Temporal Networks for Dynamic Scheduling

1st Summer School on Cognitive Robotics at MIT

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Note

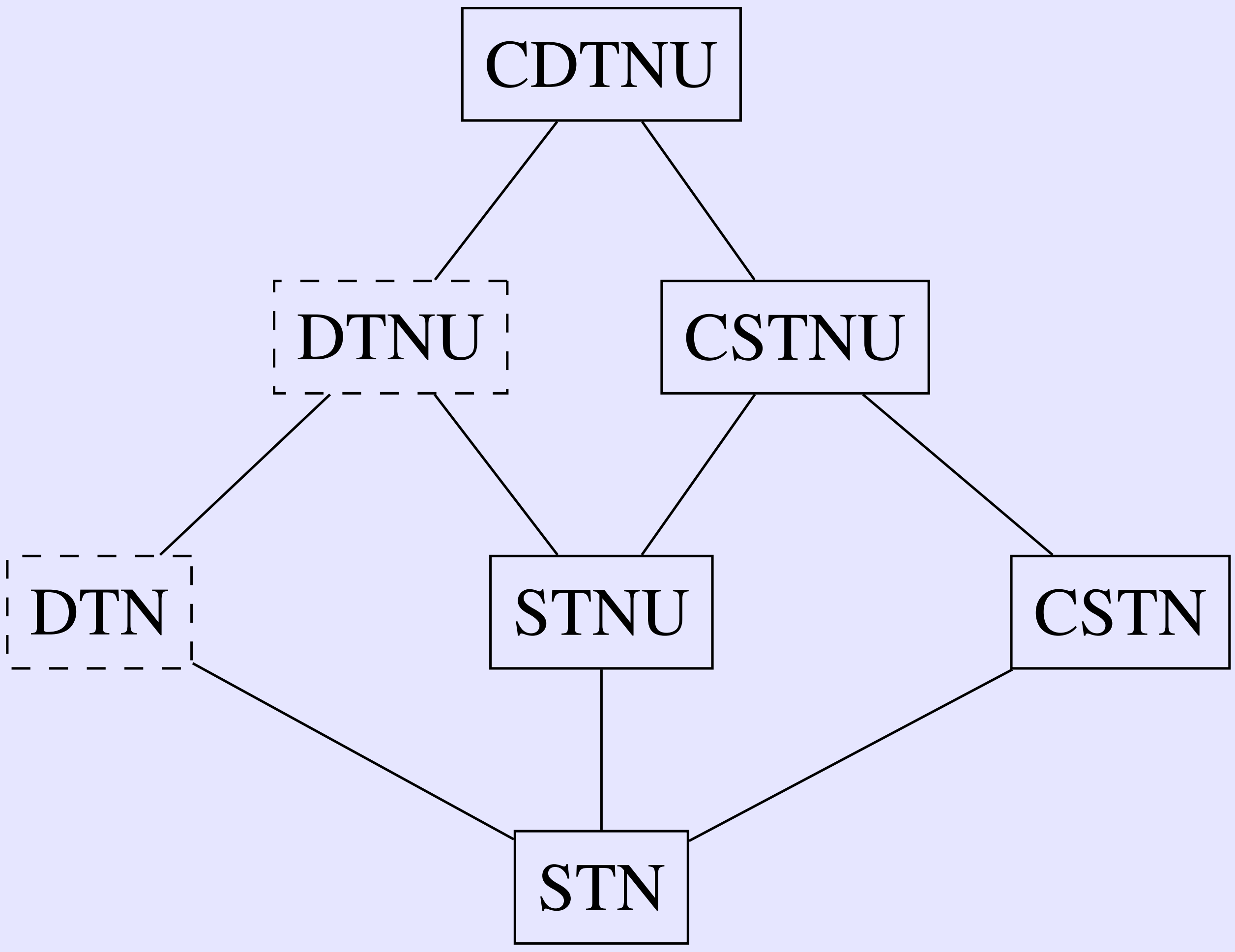
These slides have been edited from the version presented on June 12, 2017, as follows:

- Fixed the inequalities, $X - Z \leq 7$ and $Z - X \leq -3$, RE: “The *Zero* Time-Point”.
- Corrected the edge weights in the “Solving Sample STN” series.
- Inserted two slides “Remove Dominated Edges” that define “edge domination”.
- The edge from C to Z in “Making STN Dispatchable” is *not* dominated, and hence must remain in the “Dispatching the STN” series. Some edge lengths and time windows had to be corrected, too.

Outline

- Simple Temporal Networks (STNs)
- STNs with Uncertainty (STNUs)
- Conditional STNs (CSTNs)
- CSTNUs and beyond
- Conclusions

Temporal Networks



Simple Temporal Networks

Motivating Example

Goal: Fly from New York to Rome

- Leave New York after 4 p.m., June 8
- Return to New York before 10 p.m., June 18
- Away from New York no more than 7 days
- In Rome at least 5 days
- Return flight lasts no more than 7 hours

Simple Temporal Network (STN)*

- Includes time-points and temporal constraints
- Flexible: Time-points may "float"; not "nailed down" until they are *executed*
- Efficient algorithms for determining consistency, managing real-time execution, and handling new constraints

* (Dechter, Meiri, and Pearl 1991)

Simple Temporal Network*

A *Simple Temporal Network (STN)* is a pair, $\mathcal{S} = (\mathcal{T}, \mathcal{C})$, where:

- $\mathcal{T}$ is a set of time-point variables: $\{t_1, \ldots, t_n\}$; and
- $\mathcal{C}$ is a set of binary constraints, each of the form:

  $t_j - t_i \leq \delta$, where δ is a real number.

* (Dechter, Meiri, and Pearl 1991)

The *Zero* Time-Point, Z

- It is useful to have one time-point, called Z, whose value is fixed at 0.
- Binary constraints involving Z are equivalent to unary constraints:

$$X - Z \leq 7 \quad \Longleftrightarrow \quad X \leq 7$$

$$Z - X \leq -3 \quad \Longleftrightarrow \quad X \geq 3$$

Basic Notions for STNs

- A *solution* to an STN $\mathcal{S} = (\mathcal{T}, \mathcal{C})$ is a complete set of assignments to the time-points in $\mathcal{T}$:

$$\{t_1 = w_1,\ t_2 = w_2,\ \ldots,\ t_n = w_n\}$$

  that together satisfy all of the constraints in $\mathcal{C}$.

- An STN with at least one solution is *consistent.*

- STNs with identical solution sets are *equivalent*.

STN for Travel Example

$\mathcal{T} \; = \; \{Z, t_1, t_2, t_3, t_4\}, \qquad Z = \text{Noon, June 8.}$

$\mathcal{C} =$

$$\left\{ \begin{array}{rcrl} Z - t_1 & \leq & -4 & \text{(Lv NYC after 4 p.m., June 8)} \\ t_4 - Z & \leq & 250 & \text{(Av NYC by 10 p.m., June 18)} \\ t_4 - t_1 & \leq & 168 & \text{(Gone no more than 7 days)} \\ t_2 - t_3 & \leq & -120 & \text{(In Rome at least 5 days)} \\ t_4 - t_3 & \leq & 7 & \text{(Return flight less than 7 hrs)} \end{array} \right\}$$

Graph for an STN*

The *graph* for an STN, $\mathcal{S} = (\mathcal{T}, \mathcal{C})$, is a graph, $\mathcal{G} = (\mathcal{T}, \mathcal{E})$, where:

- Time-points in $\mathcal{S}$ $\iff$ nodes in $\mathcal{G}$
- Constraints in $\mathcal{C}$ $\iff$ edges in $\mathcal{E}$:

$$Y - X \leq \delta \quad \iff \quad X \xrightarrow{\delta} Y$$

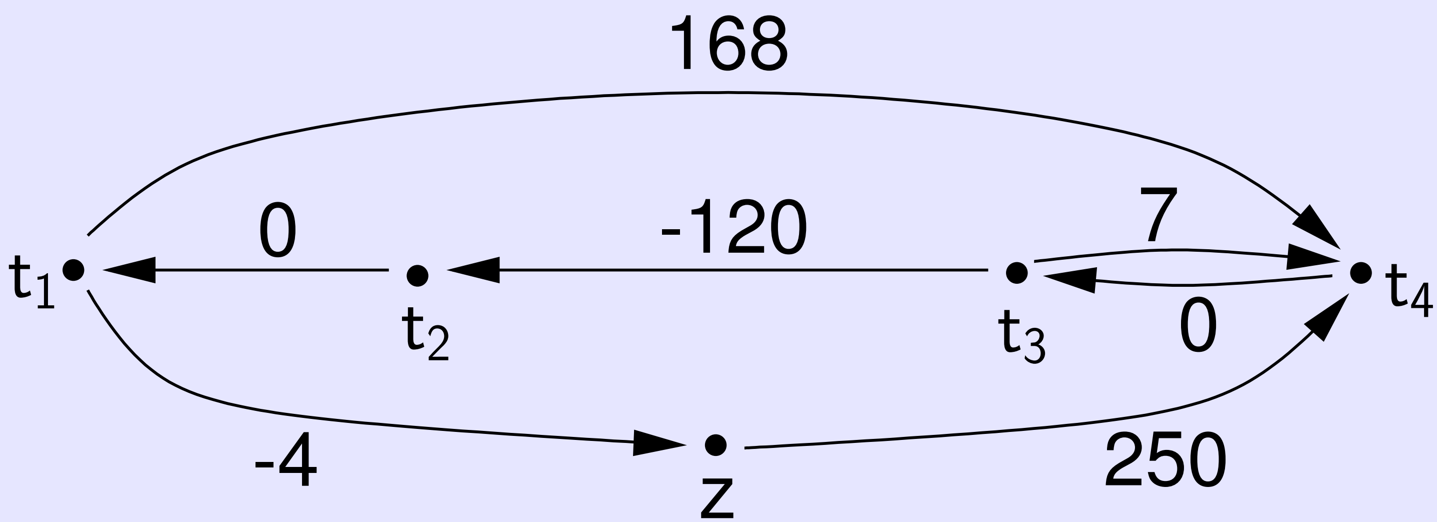
* (Dechter, Meiri, and Pearl 1991)

Graphical Representations

Constraint(s)	Edge(s)	Alt. Edge(s)
$3 \leq Y - X \leq 7$	$X \underset{-3}{\overset{7}{\rightleftarrows}} Y$	$X \xrightarrow{[3, 7]} Y$
$4 \leq X \leq 9$	$Z \underset{-4}{\overset{9}{\rightleftarrows}} X$	$Z \xrightarrow{[4, 9]} X$

Graph for Airline Scenario

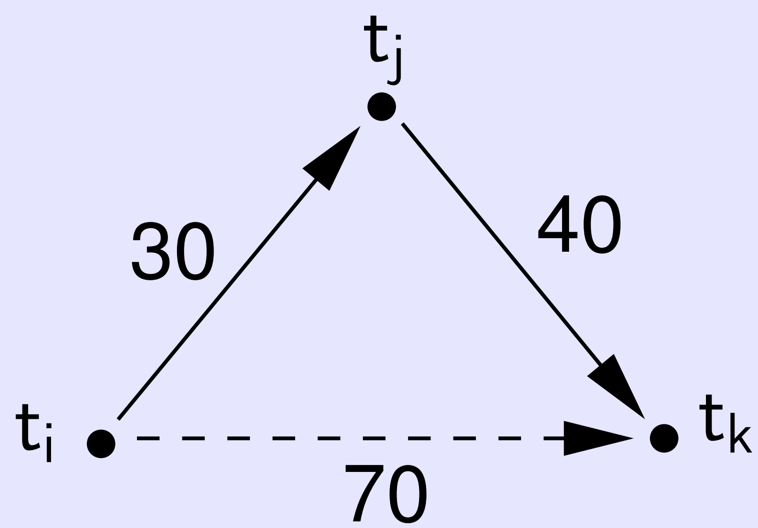
$$\left\{\begin{array}{rcrcrcr} Z - t_1 & \leq & -4, & \quad & t_4 - Z & \leq & 250 \\ t_4 - t_1 & \leq & 168, & & t_2 - t_3 & \leq & -120 \\ t_4 - t_3 & \leq & 7, & & t_1 - t_2 & \leq & 0 \\ t_3 - t_4 & \leq & 0 & & & & \end{array}\right\}$$



Implicit Constraints

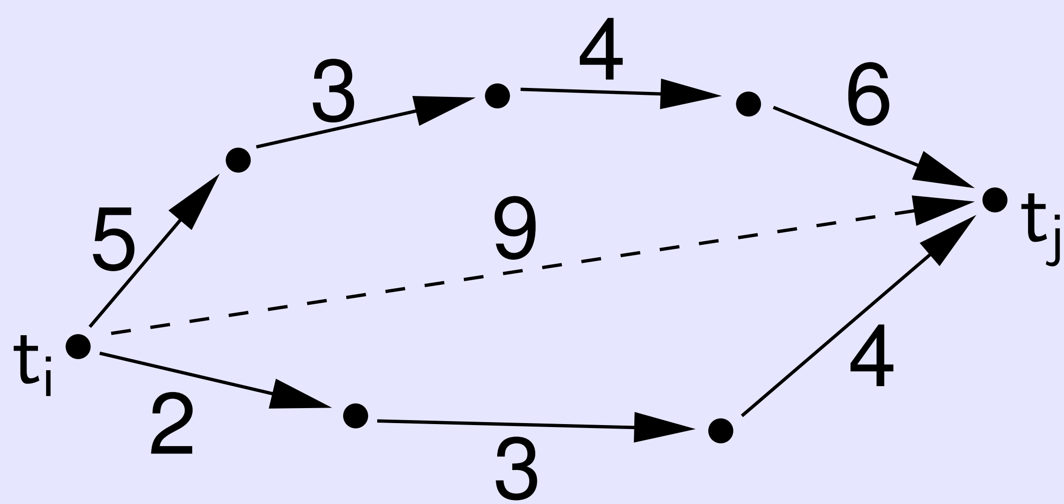
Explicit constraints combine to form implicit constraints:

$$\begin{array}{rcl} t_j - t_i & \leq & 30 \\ t_k - t_j & \leq & 40 \\ \hline t_k - t_i & \leq & 70 \end{array}$$



Chains of Constraints as Paths

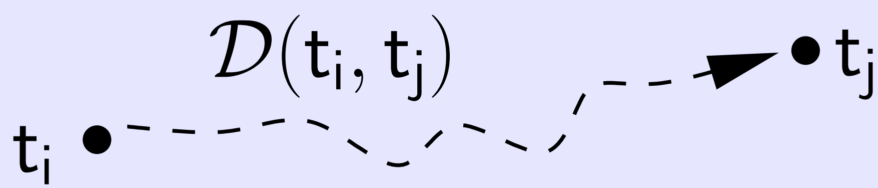
- Chains of constraints correspond to *paths* in the graph.
- *Stronger* constraints correspond to *shorter* paths.



Distance Matrix *

The *Distance Matrix* for an STN, $\mathcal{S} = (\mathcal{T}, \mathcal{C})$, is a matrix $\mathcal{D}$ defined by:

$\mathcal{D}(t_i, t_j) \;=\;$ Length of Shortest Path from t_i to t_j in the graph for $\mathcal{S}$



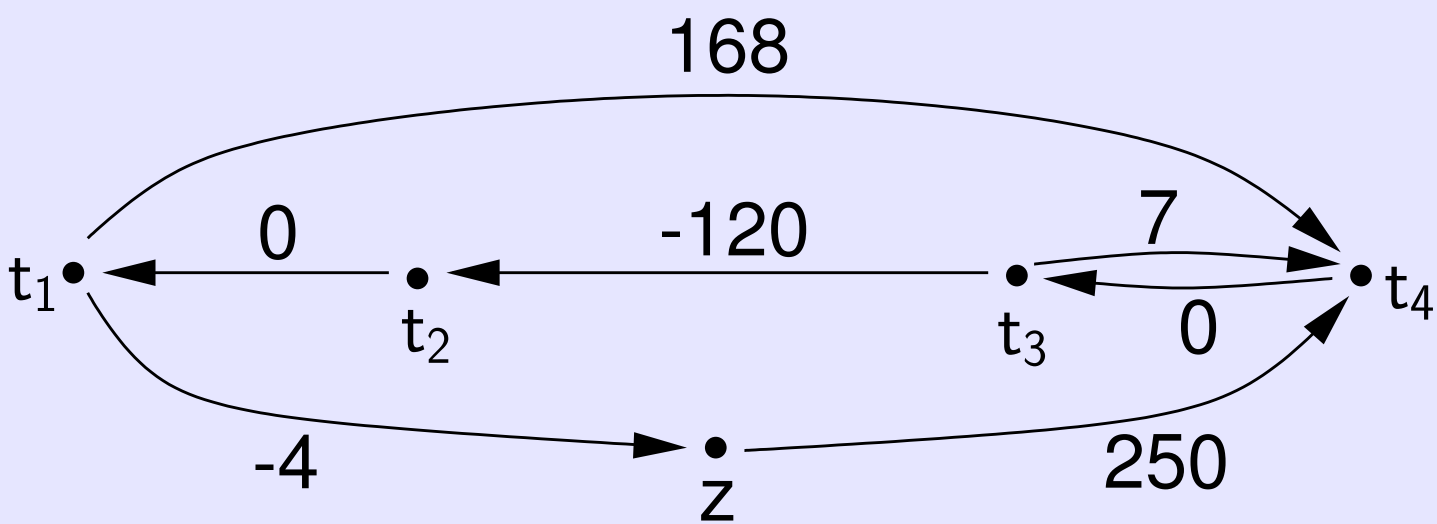
* (Dechter, Meiri, and Pearl 1991)

Distance Matrix (cont'd.)

- The strongest implicit constraint on t_i and t_j in $\mathcal{S}$ is:
  $$t_j - t_i \ \leq \ \mathcal{D}(t_i, t_j)$$
- $\mathcal{D}$ is the *All-Pairs, Shortest-Path* (APSP) Matrix for the STN's graph.*

* (Cormen, Leiserson, and Rivest 1990)

Travel Scenario's Distance Matrix



$\mathcal{D}$	z	t_1	t_2	t_3	t_4
z	0	130	130	250	250
t_1	-4	0	48	168	168
t_2	-4	0	0	168	168
t_3	-124	-120	-120	0	7
t_4	-124	-120	-120	0	0

Fundamental Theorem of STNs*

For an STN $\mathcal{S}$, with graph $\mathcal{G}$, and distance matrix $\mathcal{D}$, the following are equivalent:

- $\mathcal{S}$ is consistent
- $\mathcal{D}$ has non-negative values on main diagonal
- $\mathcal{G}$ has no negative-length loops

* (Dechter, Meiri, and Pearl 1991)

Computing $\mathcal{D}$ from Scratch

For an STN with n time-points and m edges:

- Floyd-Warshall Algorithm: $O(n^3)$
- Johnson's Algorithm: $O(n^2 \log n + nm)$

(Cormen, Leiserson, and Rivest 1990)

Dynamically Updating $\mathcal{D}$

- $O(n^2)$-time *incremental* algorithms update $\mathcal{D}$ in response to inserting a new constraint.

- $O(n^3)$-time *decremental* algorithms update $\mathcal{D}$ in response to weakening/deleting a constraint.

(Rohnert 1985; Even and Gazit 1985; Gerevini, Perini, and Ricci 1996; Ramalingam and Reps 1996; Cesta and Oddi 1996; Demetrescu and Italiano 2002)

Incremental Consistency

Verifying consistency after inserting/weakening constraints is less expensive than fully updating the distance matrix.*

- Algorithm maintains/updates a solution to the STN.
- Can verify consistency in $O(m + n \log n)$ time after inserting a new constraint.
- Deleting/weakening a constraint requires only constant time.

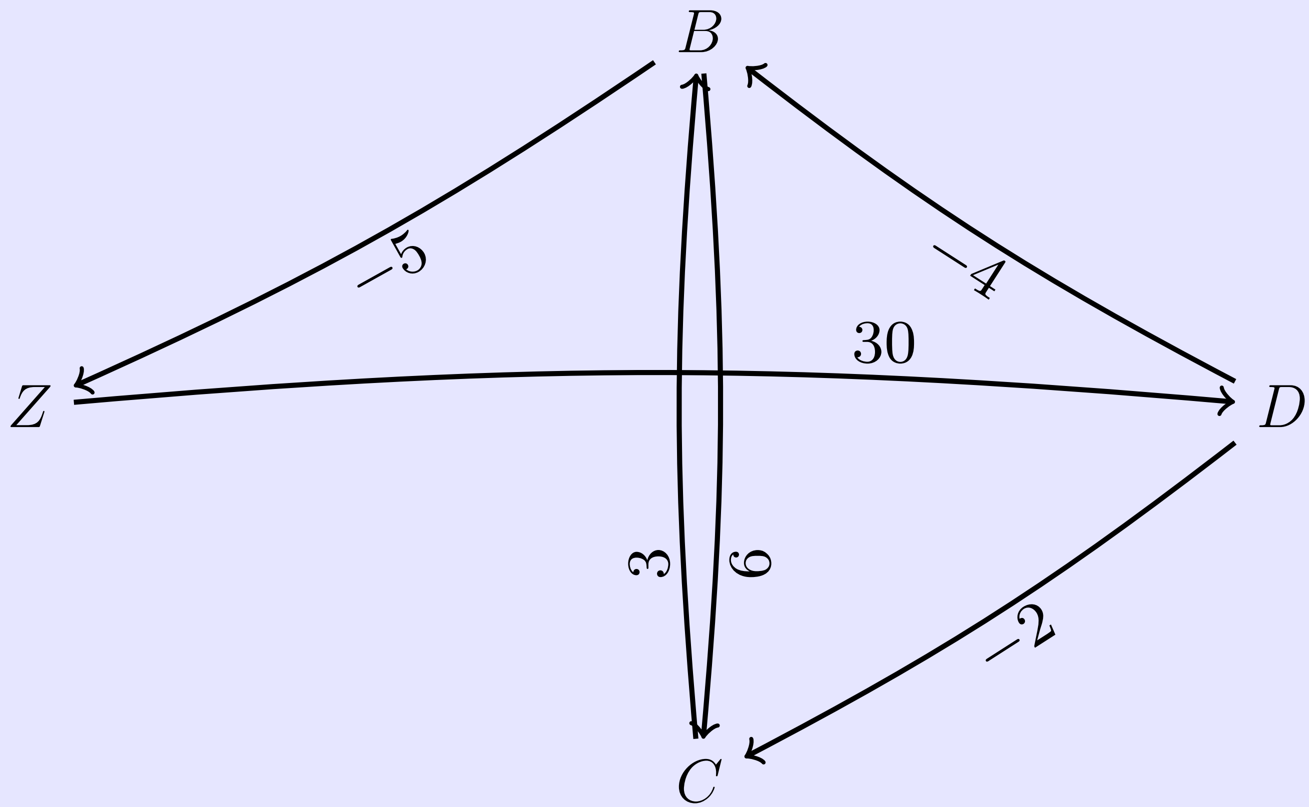
* (Ramalingam et al. 1999)

Finding a solution for an STN

- $\mathcal{D}$ has all necessary information.
- Time window for any X: $[-\mathcal{D}(X,Z),\ \mathcal{D}(Z,X)]$
- Simple algorithm to find a solution:
  - ○ Pick any time-point that doesn't yet have a value;
  - ○ Give it a value from its time-window;
  - ○ Update $\mathcal{D}$; $\Leftarrow$ expensive ...
  - ○ Repeat until all time-points have values.

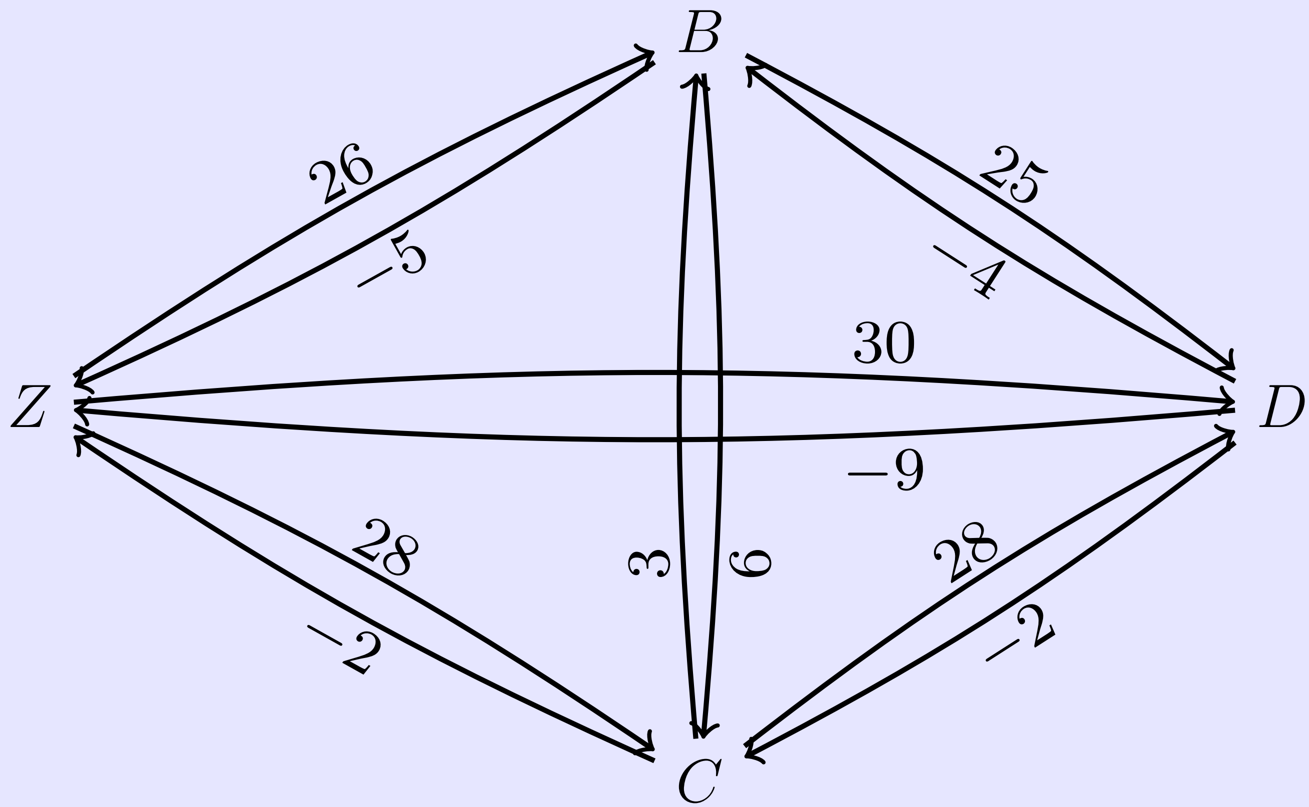
* (Dechter, Meiri, and Pearl 1991)

Sample STN



“Solving” Sample STN

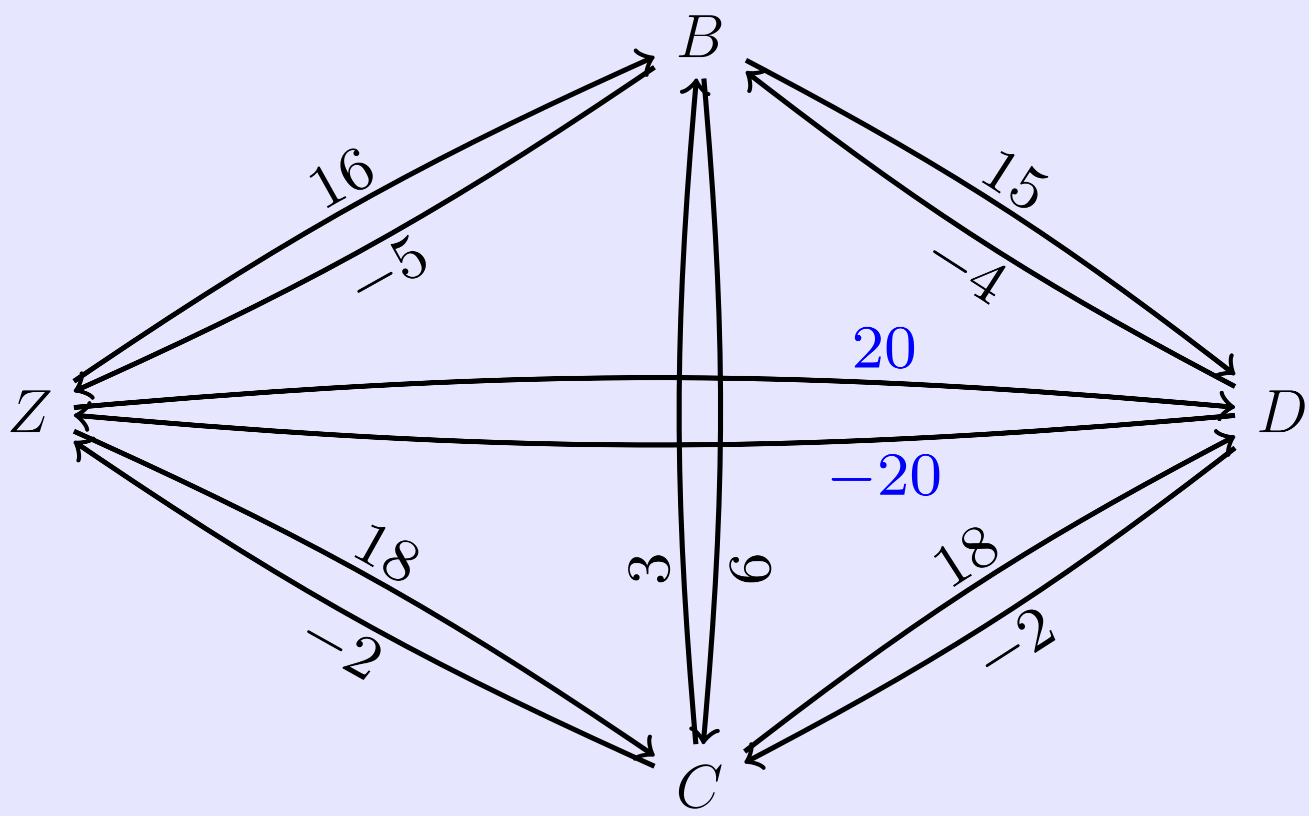
First, form APSP graph (equiv. compute $\mathcal{D}$).



Time Windows: $B \in [5, 26],\ C \in [2, 28],\ D \in [9, 30]$

“Solving” Sample STN (ctd.)

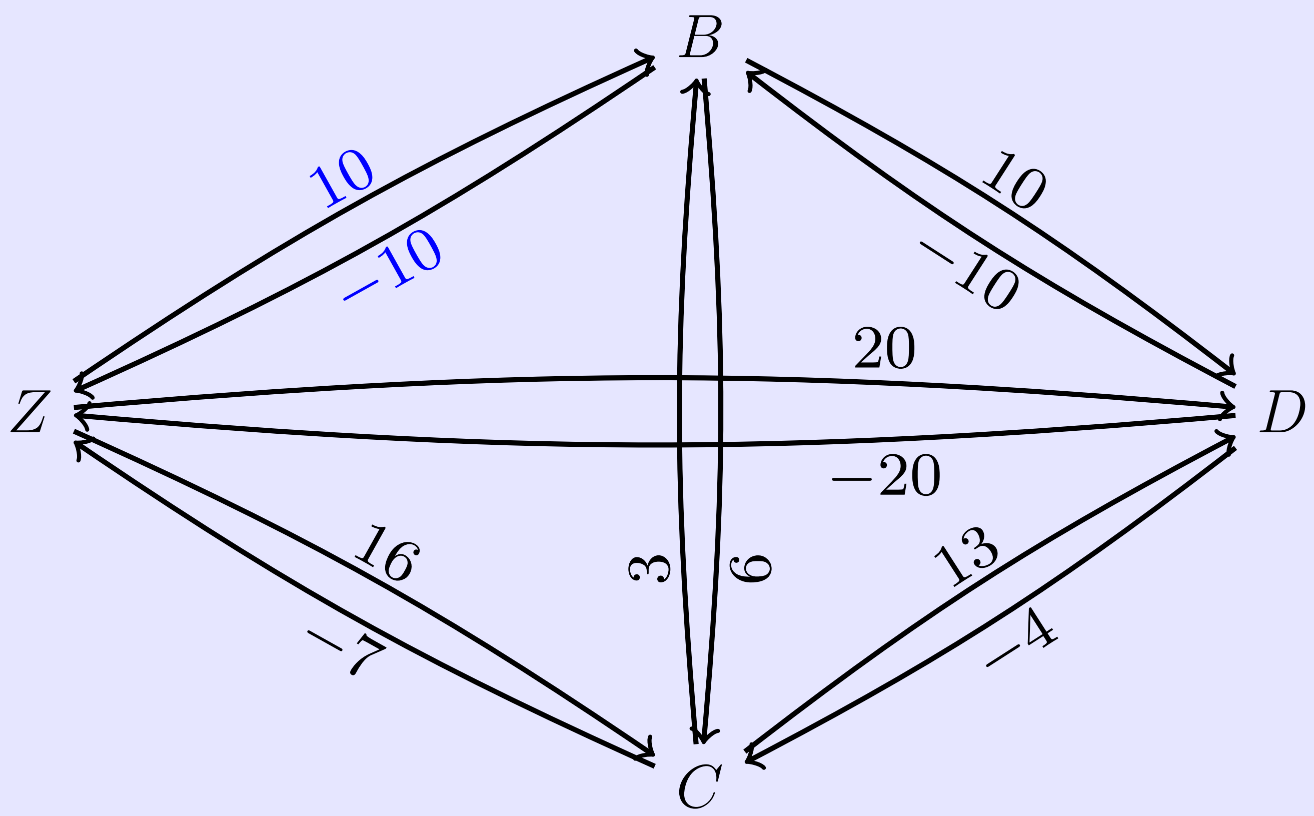
Next, select $D = 20$; and update APSP graph:



Remaining Time Windows: $B \in [5, 16],\ C \in [2, 18]$

“Solving” Sample STN (ctd.)

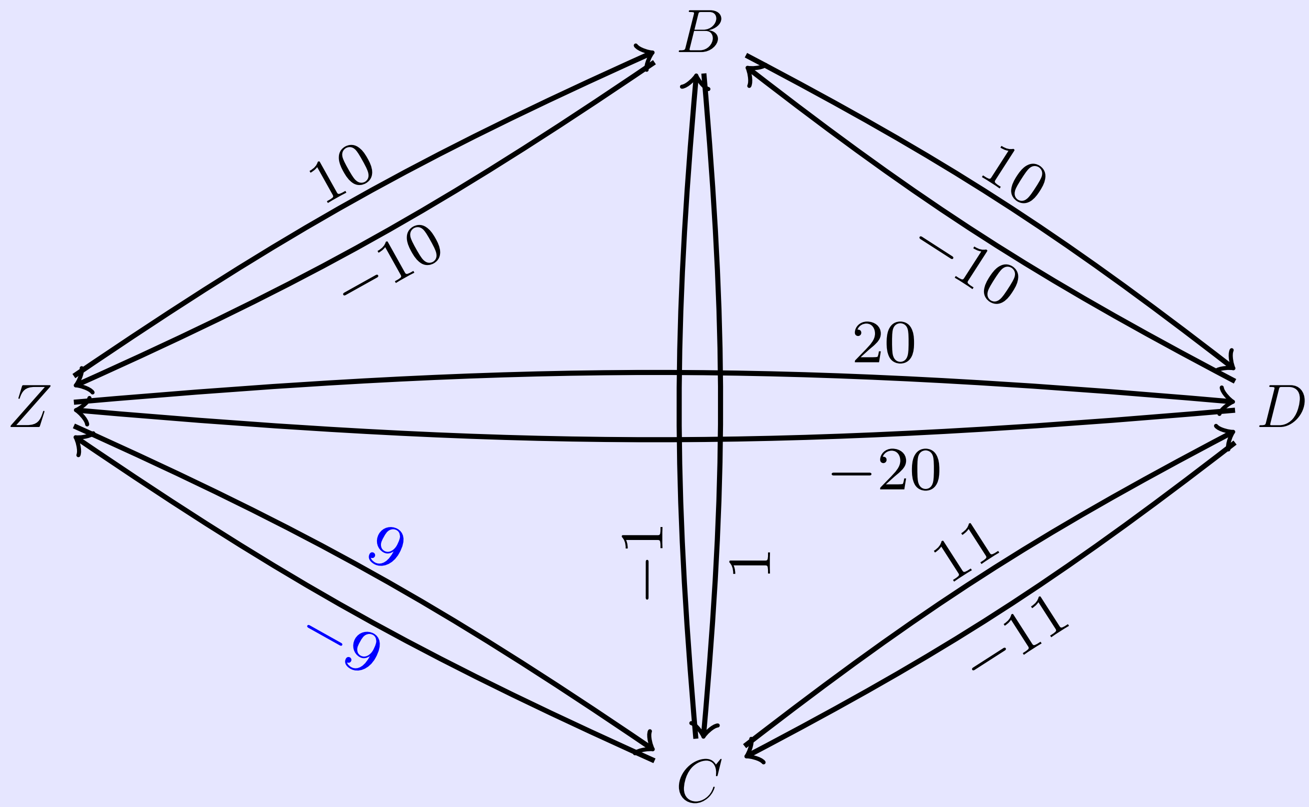
Next, select $B = 10$; and update APSP graph:



Remaining Time Windows: $C \in [7, 16]$

“Solving” Sample STN (ctd.)

Finally, select $C = 9$; and update APSP graph:



Easy to verify that this is a solution.

Problems with "Solving" an STN

- May need to go back in time:
  Pick $D = 20$, then after updating, pick $B = 10$
  (i.e., no relationship to real-time execution)

- Expensive to update $\mathcal{D}$

Executing an STN in real time

- Only executed *enabled* time-points: those having no negative edges to unexecuted time-points.

- Focus updating on entries involving Z: reduces cost to linear time per update, $O(n^2)$ overall.*

- Alternatively, prior to execution, transform STN into *dispatachable* form in $O(n^2 \log n + nm)$ time; then during execution, only need to propagate bounds to *neighboring* time-points.†

*(Hunsberger 2008); †(Muscettola, Morris, and Tsamardinos 1998), †(Tsamardinos, Muscettola, and Morris 1998)

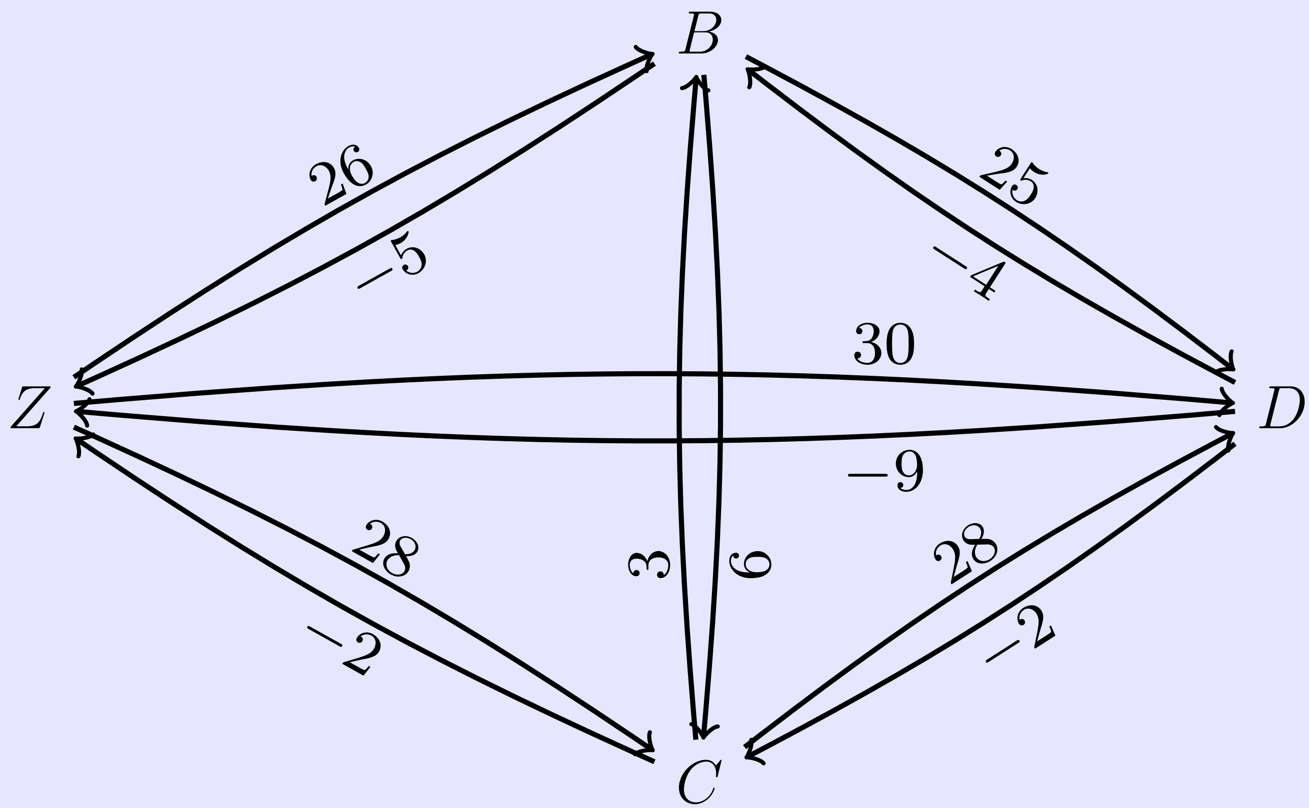
Dispatchable STN

An STN $\mathcal{S}$ is *dispatchable* if the following algorithm necessarily successfully executes $\mathcal{S}$:

1. Let $t = 0$ (current time); $\mathcal{X} = \{\}$ (executed);
   $\mathbf{E} = \{Z\}$ (currently enabled);
2. Pick any $X \in \mathbf{E}$ such that t is in X's time window;
3. Set $X := t$, and add X to $\mathcal{X}$;
4. Propagate $t \leq X \leq t$ to X's *immediate neighbors*;
5. Put into $\mathbf{E}$ all time-points Y such that all negative edges emanating from Y have a destination in $\mathcal{X}$;
6. Wait until t has advanced to some time between $\min\{lb(W) \mid W \in \mathbf{E}\}$ and $\min\{ub(W) \mid W \in \mathbf{E}\}$;
7. Repeat until all time-points are in $\mathcal{X}$ (executed).

Making STN Dispatchable

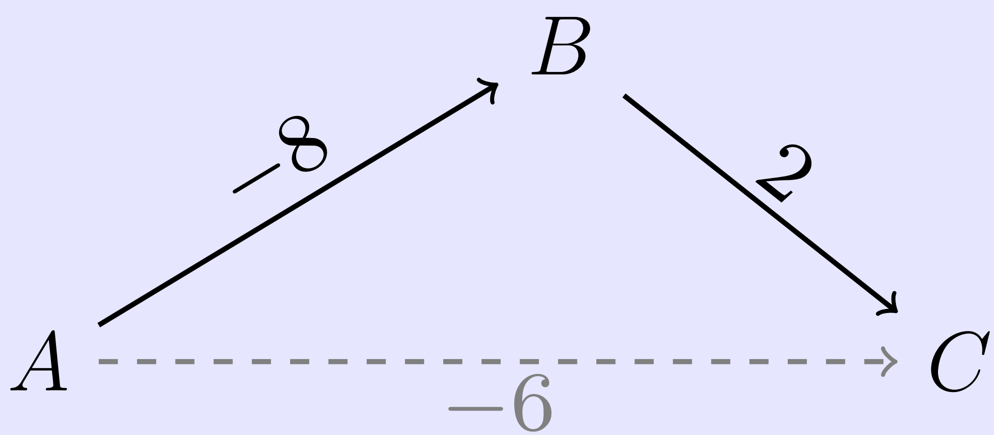
Start with APSP Graph:



Then remove *dominated* edges …

Remove Dominated Edges

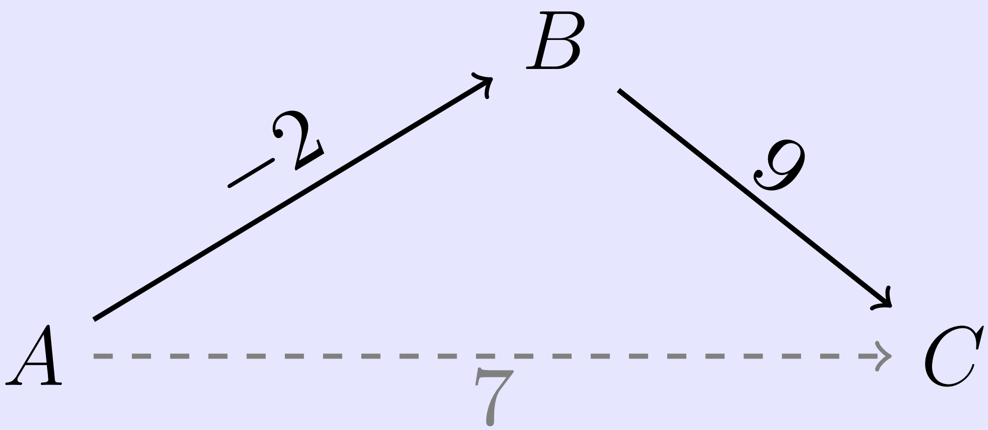
A negative edge AC is dominated by a negative edge AB if $\mathcal{D}(A,B) + \mathcal{D}(B,C) = \mathcal{D}(A,B)$:



Note: AB and AC have the *same source* node: A.

Remove Dominated Edges (ctd.)

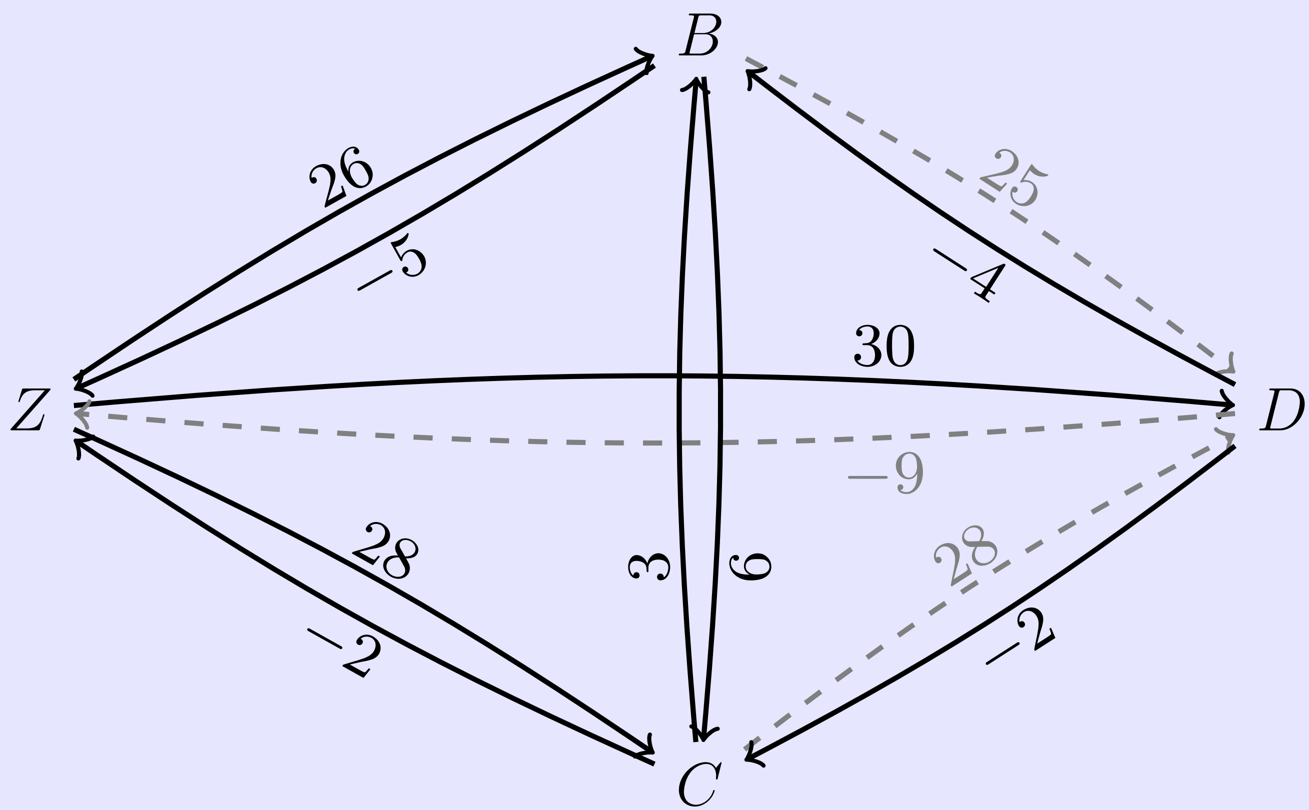
A non-negative edge AC is dominated
by a non-negative edge BC
if $\mathcal{D}(A,B) + \mathcal{D}(B,C) = \mathcal{D}(A,B)$:



Note: BC and AC have the *same destination* node: A.

Making STN Dispatchable (ctd.)

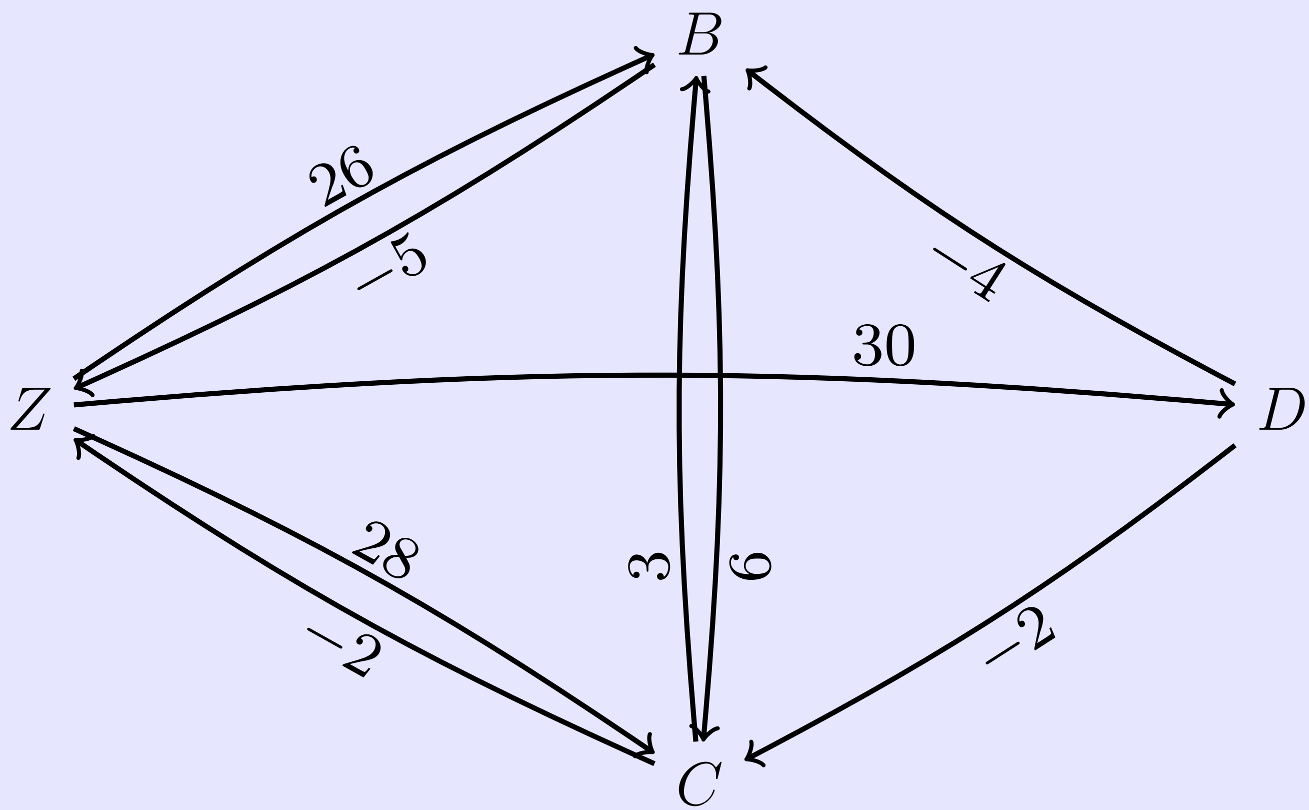
Remove "dominated" edges:*



*(Muscettola, Morris, and Tsamardinos 1998)

Dispatching the STN

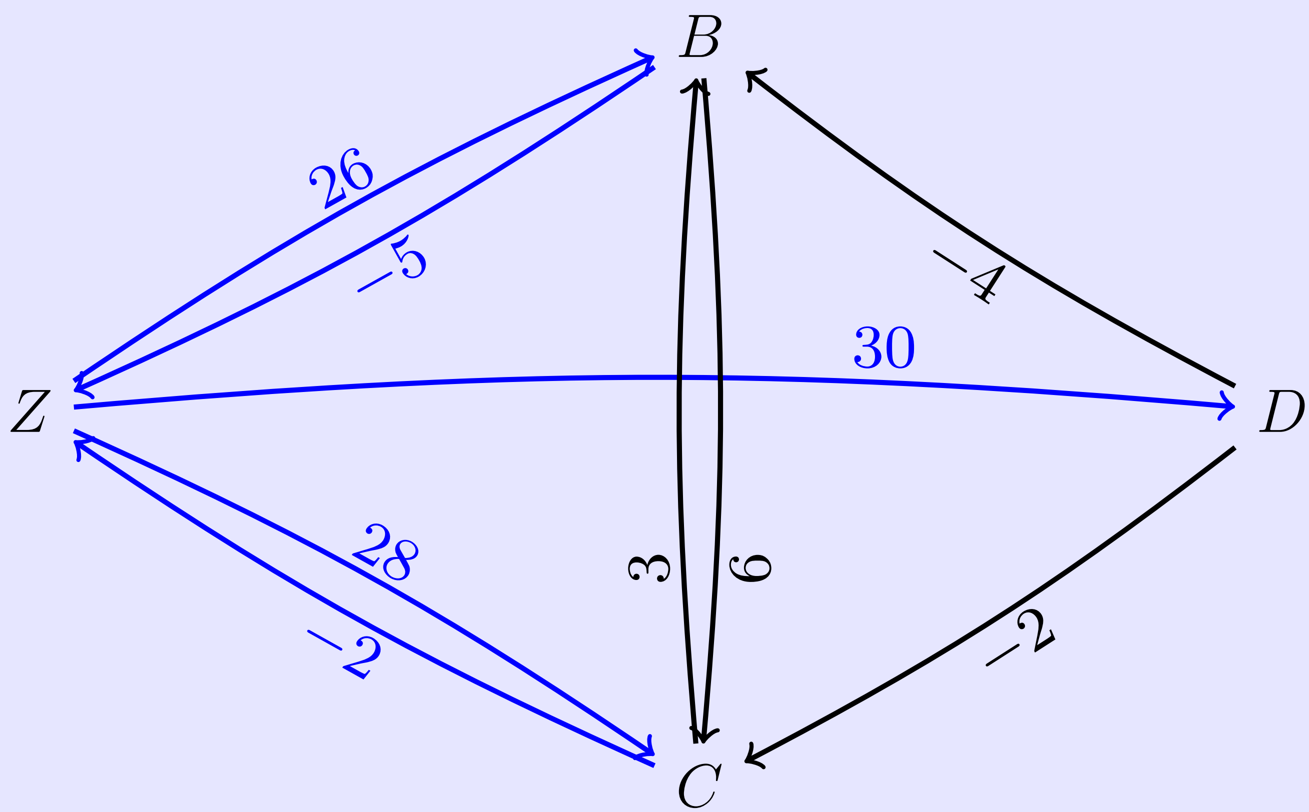
Initially: $t = 0$, $\mathcal{X} = \{\}$, $\mathbf{E} = \{Z\}$.



Pick Z from $\mathbf{E}$. Set $Z = 0$.

Dispatching the STN (ctd.)

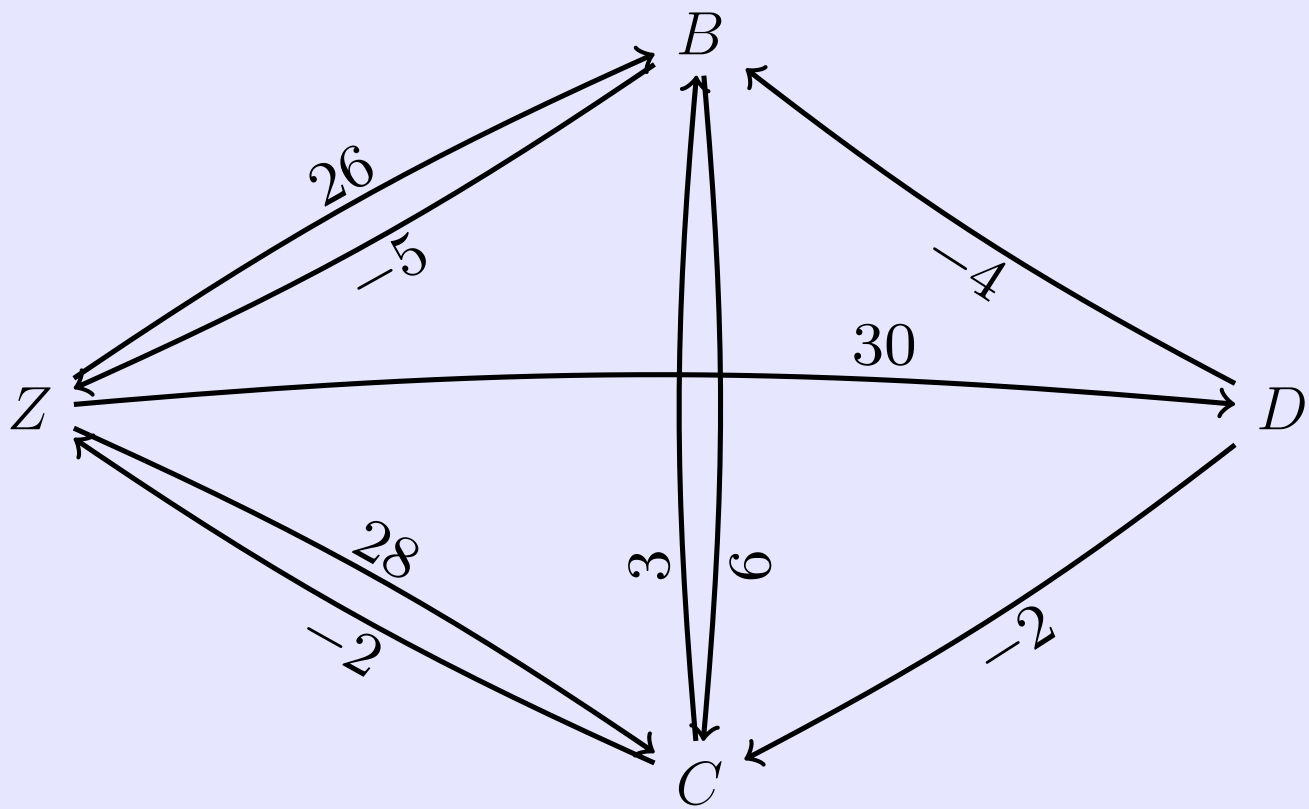
Propagate $Z = 0$ to neighbors;



$$\mathcal{X} = \{Z\}, \mathbf{E} = \{B, C\}; B \in [5, 26], C \in [2, 28], D \in [0, 30].$$

Dispatching the STN (ctd.)

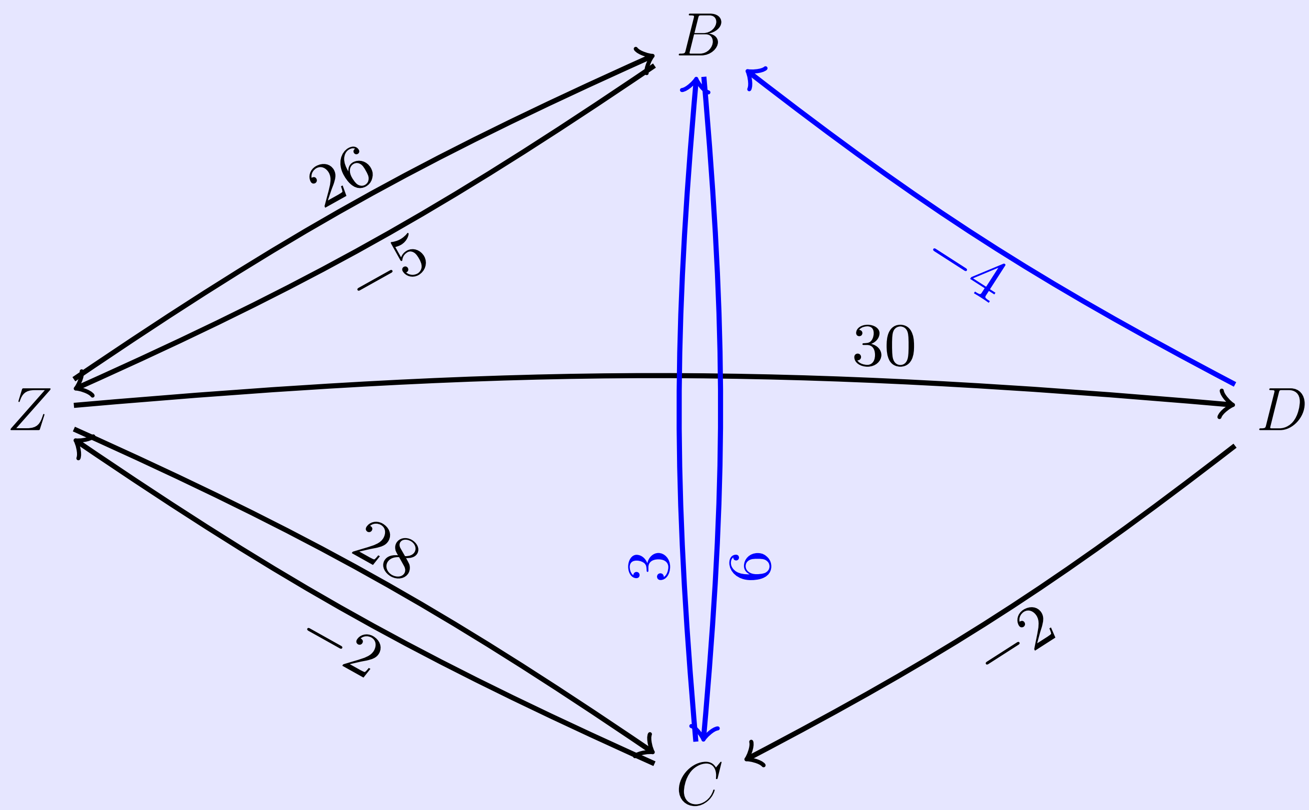
$\mathcal{X} = \{Z\}$, $\mathbf{E} = \{B, C\}$; Bounds: $B \in [5, 26]$, $C \in [2, 28]$.



Let t advance to 12; Pick B from $\mathbf{E}$; Set $B = 12$.

Dispatching the STN (ctd.)

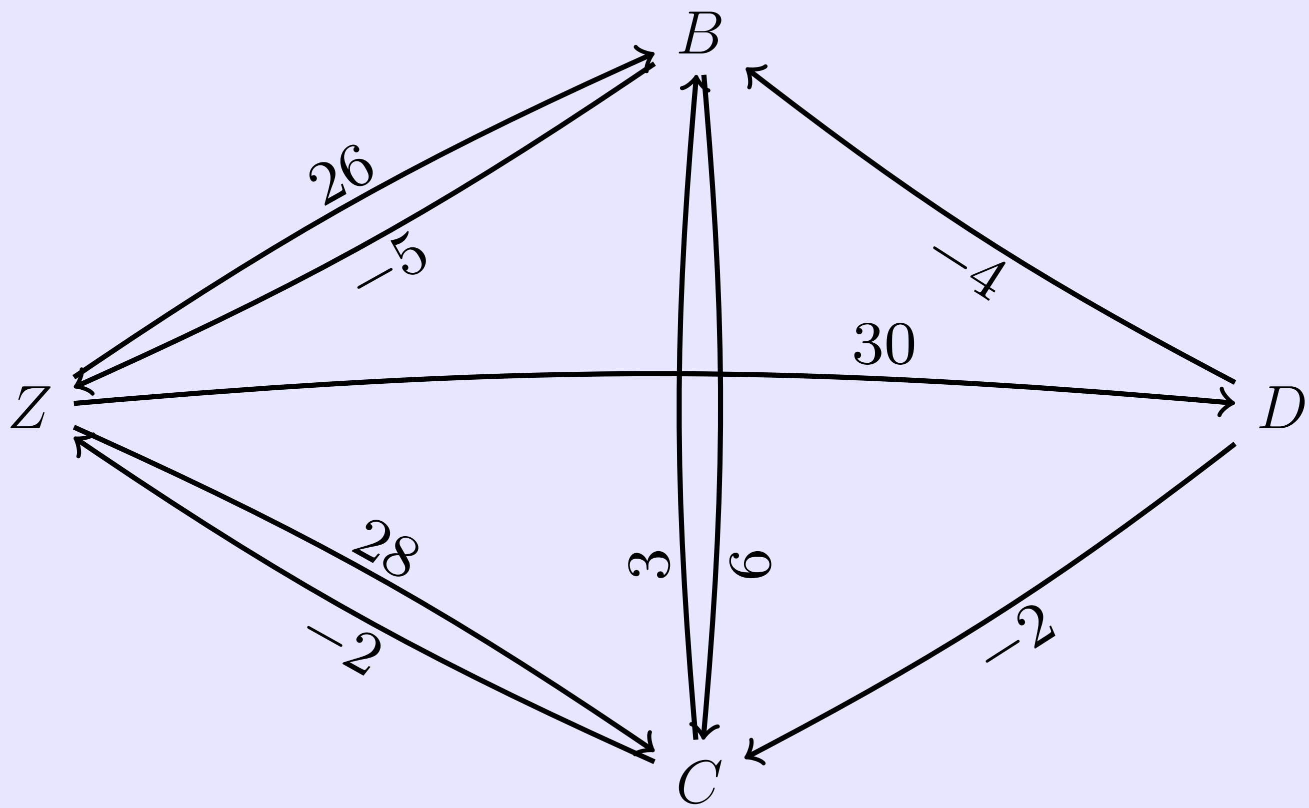
Propagate $B = 12$ to neighbors



$\mathcal{X} = \{Z, B\}, t = 12, \mathbf{E} = \{C\}, C \in [12, 18], D \in [16, 30]$

Dispatching the STN (ctd.)

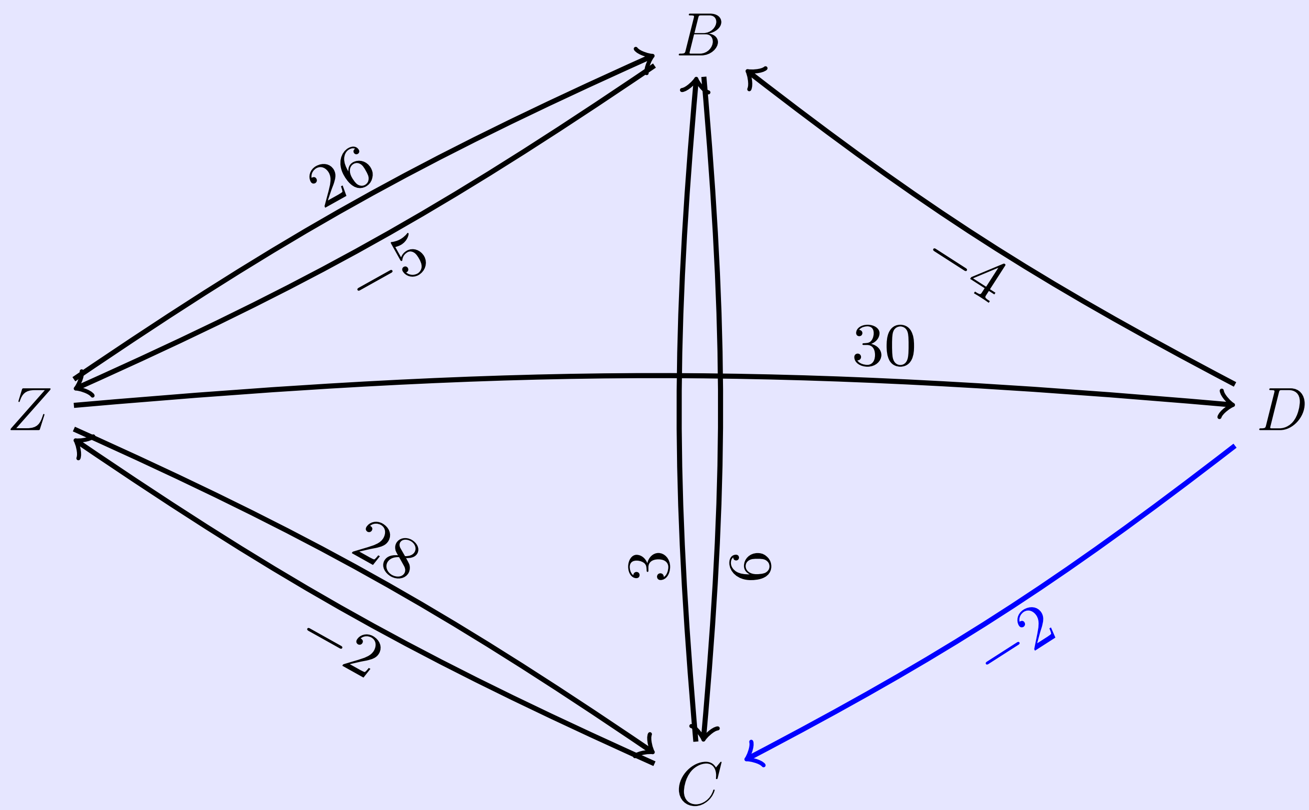
$\mathcal{X} = \{Z, B\}, t = 12, \mathbf{E} = \{C\}, C \in [12, 18], D \in [16, 30]$



Let t advance to 16, pick C from $\mathbf{E}$, set $C = 16$.

Dispatching the STN (ctd.)

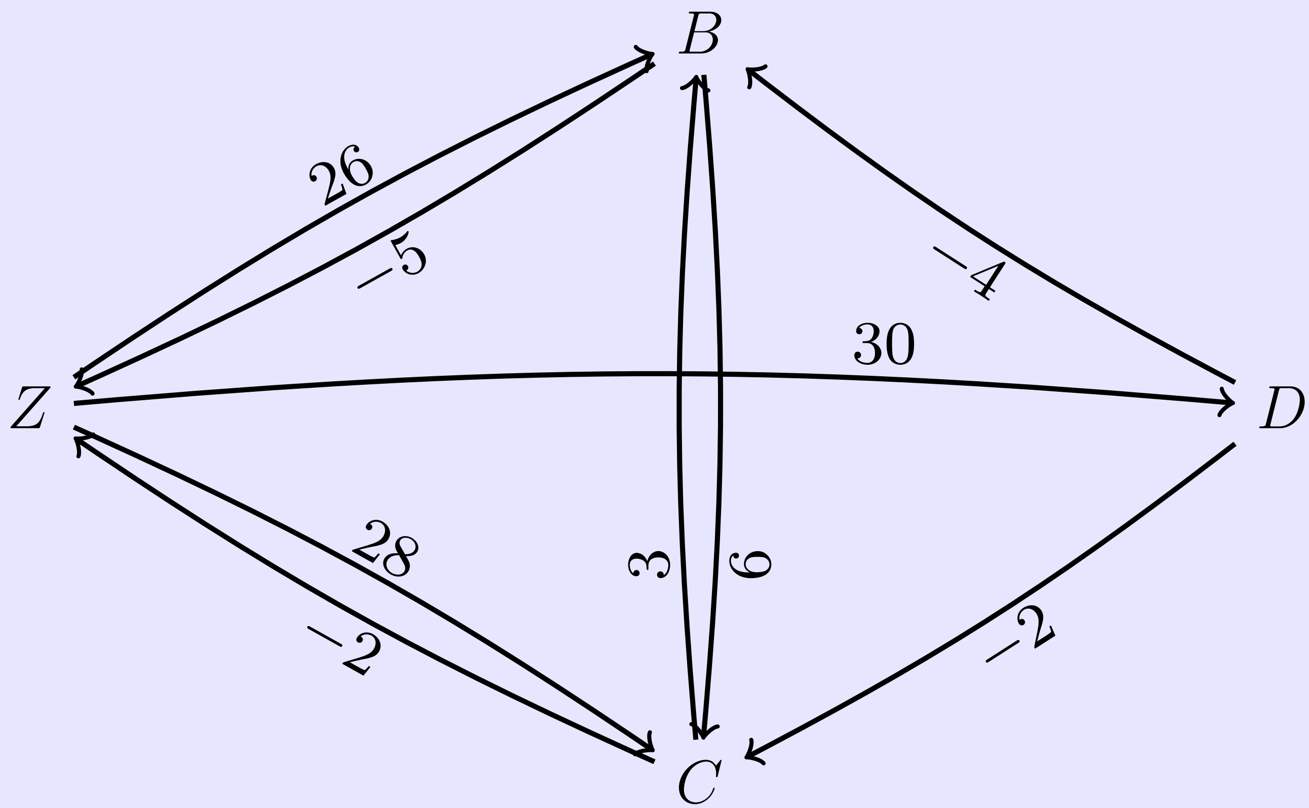
Propagate $C = 16$ to C's only remaining neighbor, D.



$$\mathcal{X} = \{Z, B, C\}, t = 16, \mathbf{E} = \{D\}, D \in [18, 30]$$

Dispatching the STN (ctd.)

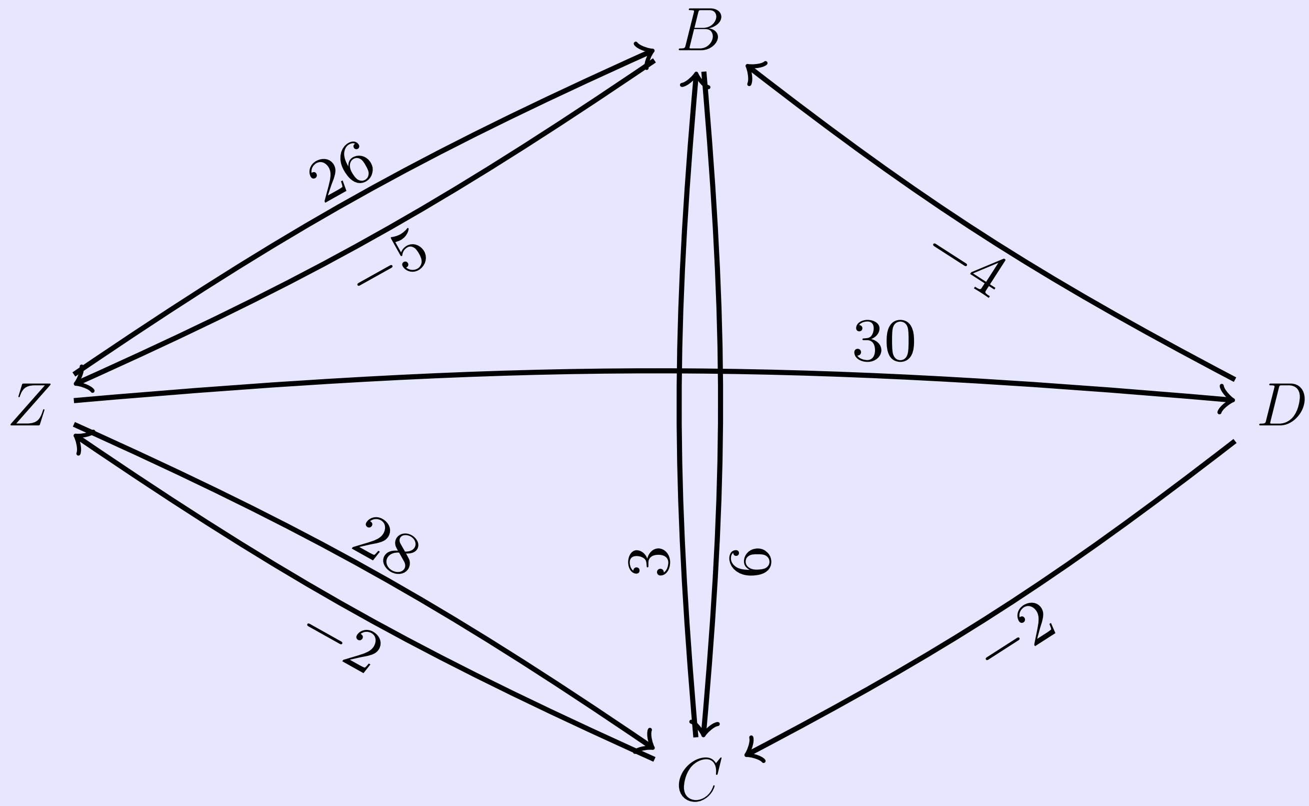
$\mathcal{X} = \{Z, B, C\}$, $t = 16$, $\mathbf{E} = \{D\}$, $D \in [18, 30]$



Let t advance to 25, pick D from $\mathbf{E}$, set $D = 25$.

Dispatching the STN (ctd.)

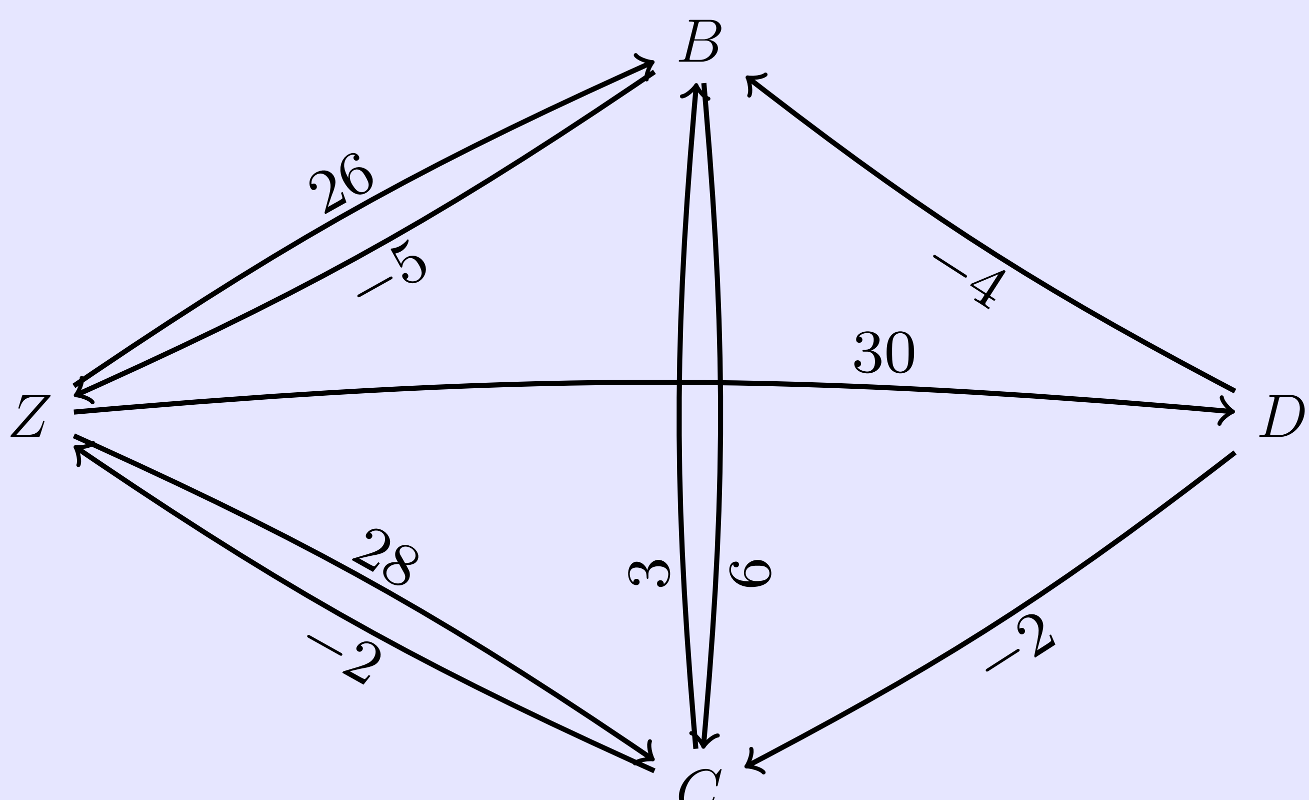
$$\mathcal{X} = \{Z, B, C, D\}, t = 25, \mathbf{E} = \{\}$$



Solution: $Z = 0, B = 12, C = 16, D = 25.$

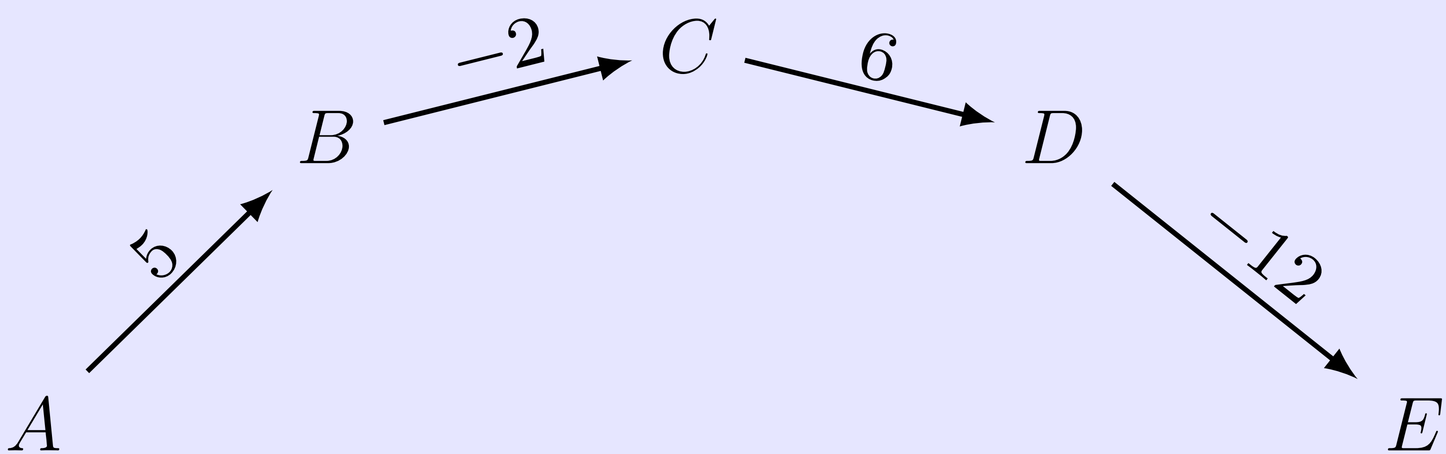
Dispatching the STN (ctd.)

Easy to check that $Z = 0, C = 20, B = 23, D = 28$ can also be generated by the dispatcher.



New View of Dispatchability*

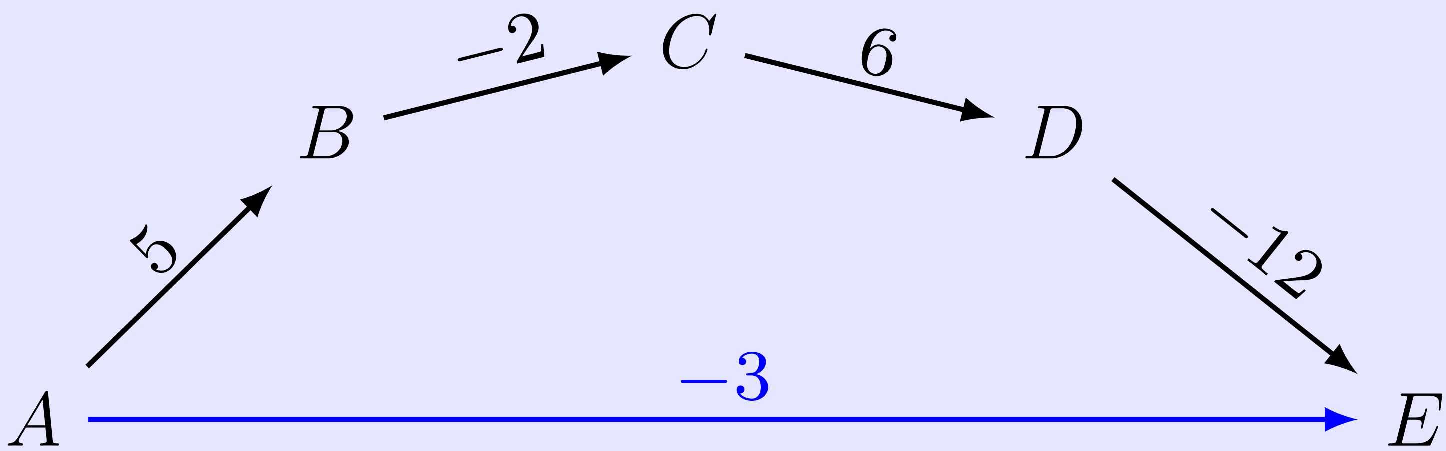
(1) A path $\mathcal{P}$ has the prefix/postfix property if every proper prefix of $\mathcal{P}$ has non-negative length, and every proper postfix of $\mathcal{P}$ has negative length.



* (Morris 2014)

New View of Dispatchability (ctd)

(2) An STN is PP-complete if for each shortest path from any A to any B that has the prefix/postfix property, there is an edge from A to B with the same length.



(3) A consistent and PP-complete STN is dispatchable.

More on Dispatchability

Morris (Morris 2016) presented further graphical analyses of the dispatchability of STNs.

STN Summary

- STNs have been used to provide flexible planning and scheduling systems for more than a decade.
- Efficient algorithms for checking consistency, incrementally updating the APSP matrix, and managing execution in real time for maximum flexibility.
- However, STNs cannot represent uncertainty (e.g., actions with uncertain durations) or conditional constraints (e.g., only do X if test result is negative).

STNs with Uncertainty

Motivation for STNUs

- You may control when an action starts, but not how long it takes to complete: taxi ride, bus ride, baseball game, medical procedure.
- Although their durations may be uncertain, they are often within known bounds.
- Such actions can be represented by *contingent links* in a temporal network …

STN with Uncertainty*

An STNU is a triple, $\mathcal{S} = (\mathcal{T}, \mathcal{C}, \mathcal{L})$ where:

- $\mathcal{T}$ and $\mathcal{C}$ as in an STN
- $\mathcal{L}$ — Contingent Links: (A, ℓ, u, C)
  - A is the activation time-point.
  - C is the contingent time-point.
  - Duration bounded: $C - A \in [\ell, u]$

    — but *uncontrollable*

* (Morris, Muscettola, and Vidal 2001)

STNU Graph

- Nodes and Edges as in an STN graph

$$Y - X \in [3, 7] \qquad \Longleftrightarrow \qquad \begin{tikzcd} X \arrow[r, bend left, "7"] & Y \arrow[l, bend left, "-3"] \end{tikzcd}$$

- Contingent Links $\Longleftrightarrow$ Labeled Edges*

$$C - A \in [3, 7] \qquad \Longleftrightarrow \qquad \begin{tikzcd} A \arrow[r, bend left, "c\,:\,3"] & C \arrow[l, bend left, "C\,:\,-7"] \end{tikzcd}$$

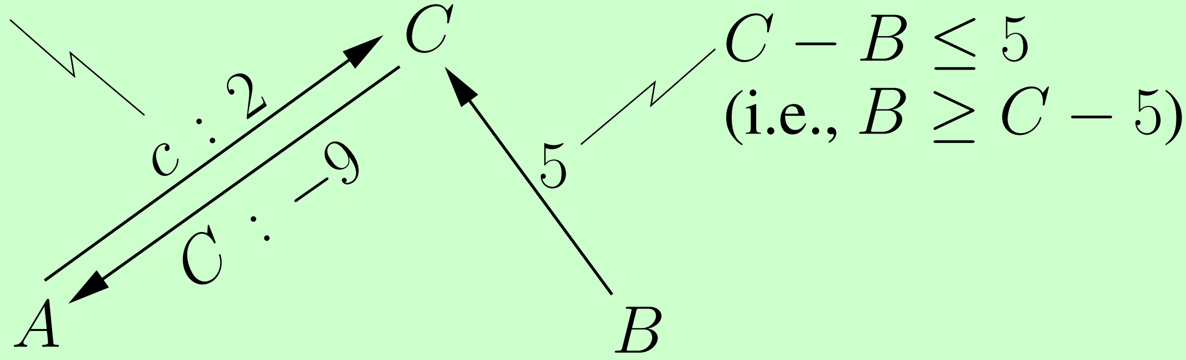
Labeled edges represent uncontrollable possibilities.

* (Morris and Muscettola 2005)

STNU Example

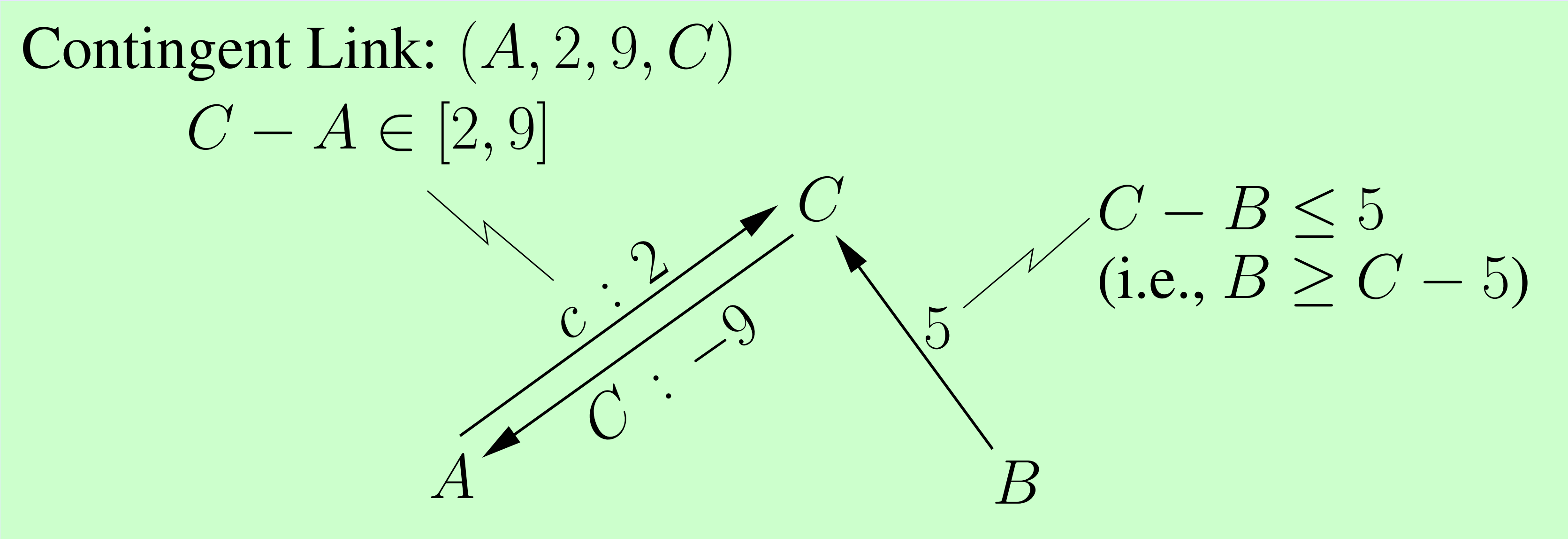
Contingent Link: $(A, 2, 9, C)$

$C - A \in [2, 9]$



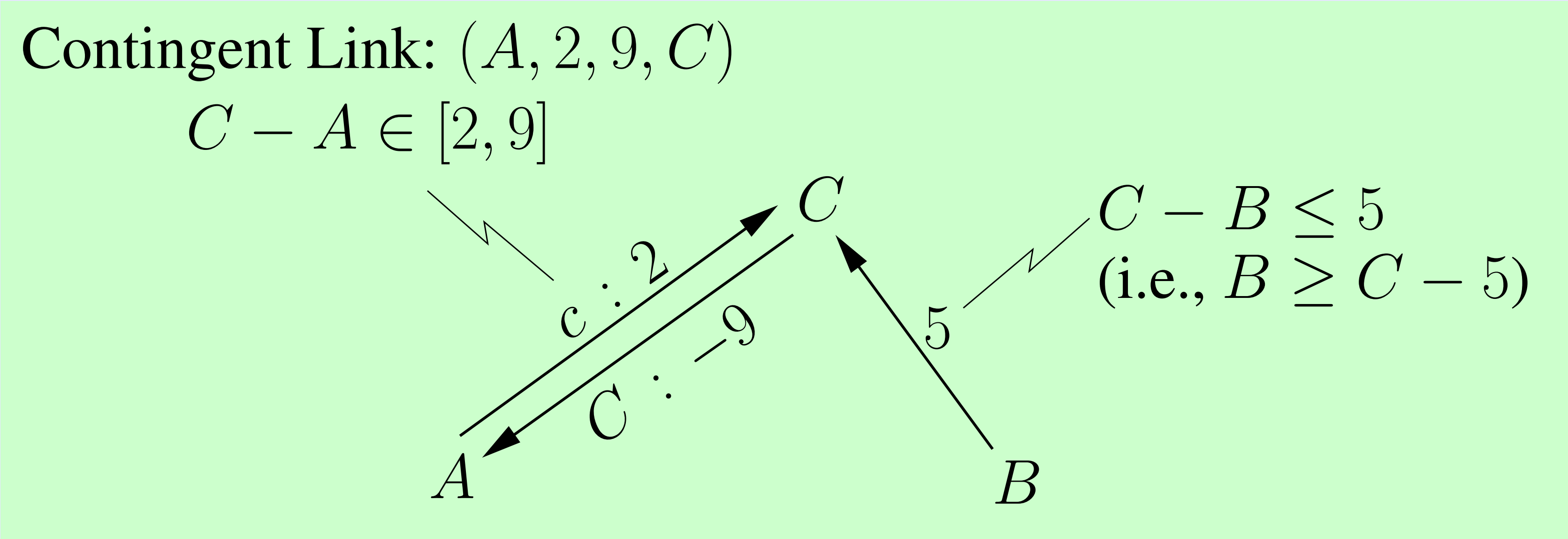
If $A = 0$, when is it safe to execute B?

STNU Example



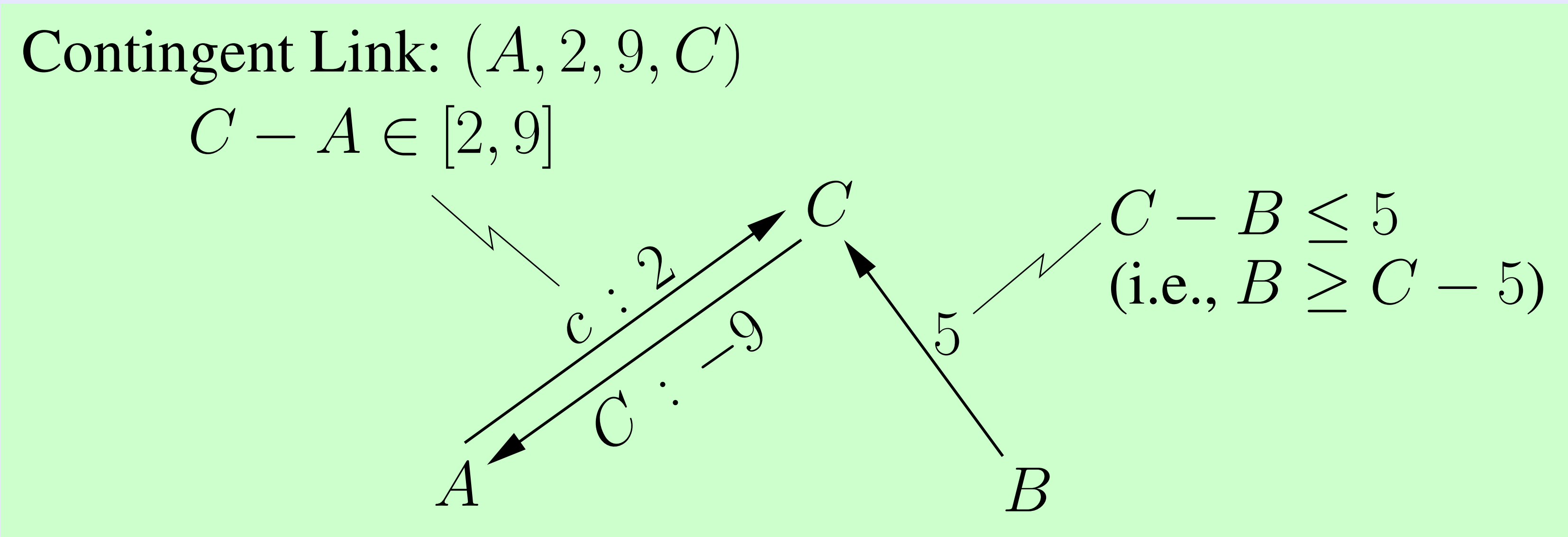
If $A = 0$ and $B = 2$, then problem if $C > 7$.

STNU Example



If $A = 0$ and $B \geq 4$, then no problems!

STNU Example



If $A = 0$ and $C = 3$, then $B > 3$ no problem!

Dynamic Controllability (DC)

An STNU is *dynamically controllable* (DC) if:

there exists a *dynamic strategy* ...

for executing the *non-contingent* time-points ...

such that *all* of the constraints will be satisfied ...

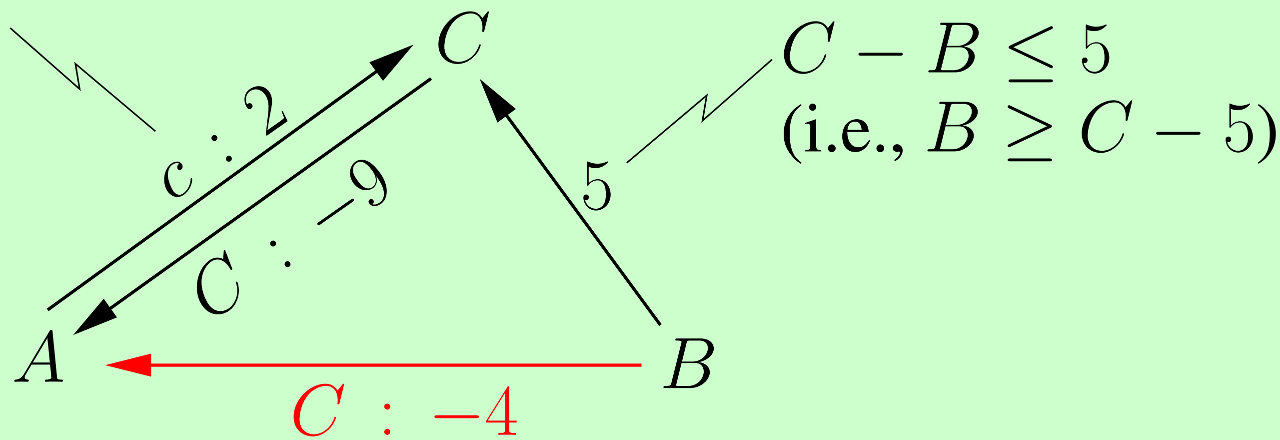
no matter how the contingent durations turn out.

$\Rightarrow$ A dynamic strategy can *react* to contingent executions.

STNU Example

Contingent Link: $(A, 2, 9, C)$

$C - A \in [2, 9]$



Strategy: As long as C unexecuted,
B must wait at least 4 after A.

Semi-Reducible Paths

- Whereas shortest paths in an STN graph represent the strongest constraints that a consistent execution must satisfy, the shortest *semi-reducible* paths in an STNU graph represent the strongest constraints that an execution strategy for an STNU must satisfy.
- The *All-Pairs, Shortest Semi-Reducible Paths* (APSSRP) matrix $\mathcal{D}^*$ for an STNU is analogous to the APSP matrix for an STN.

Fundamental Theorem of STNUs

For an STNU $\mathcal{S}$, with graph $\mathcal{G}$, and APSSRP matrix $\mathcal{D}^*$, the following are equivalent:

- $\mathcal{S}$ is dynamically controllable
- $\mathcal{G}$ has no *semi-reducible* negative loops
- $\mathcal{D}^*$ has non-negative values on its main diagonal

(Morris and Muscettola 2005; Morris 2006; Hunsberger 2010; 2013b)

DC Checking for STNUs

The worst-case time for DC-checking algorithms for STNUs has improved dramatically in recent years:

- Pseudo-polynomial: (Morris et al., 2001)
- $O(N^5)$: (Morris and Muscettola 2005)
- $O(N^4)$: (Morris 2006)
- $O(N^3)$: (Morris 2014)

And *flexibly* executing a DC STNU can be done in $O(N^3)$ time overall (Hunsberger 2013a; 2015) (Morris 2014).

Real-Time Execution Decisions*

The semantics for dynamic controllability can be stated in terms of *Real-Time Execution Decisions* (RTEDs):

- WAIT:
  Wait for some activated contingent link to complete.

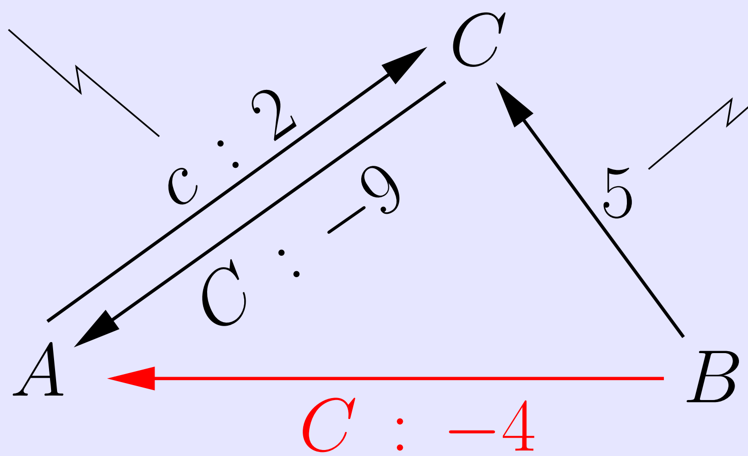
- (t, χ):
  If nothing happens before time $t \in \mathbb{R}$, then execute the (non-contingent) time-points in χ at time t.

* (Hunsberger 2009)

RTED Example

Contingent Link: $(A, 2, 9, C)$

$C - A \in [2, 9]$



$C - B \leq 5$

(i.e., $B \geq C - 5$)

Initial Decision: $(4, \{B\})$

(If nothing happens before time 4, execute B at 4.)

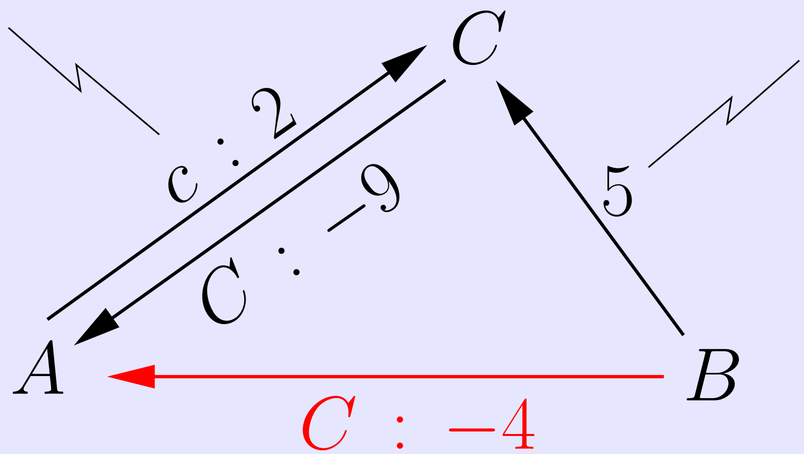
RTED Example (ctd.)

Contingent Link: $(A, 2, 9, C)$

$C - A \in [2, 9]$

$C - B \leq 5$

(i.e., $B \geq C - 5$)



Possible Outcome: C executes at time 2.

Next decision: $(3, \{B\})$

(If nothing happens before time 3, execute B at 3.)

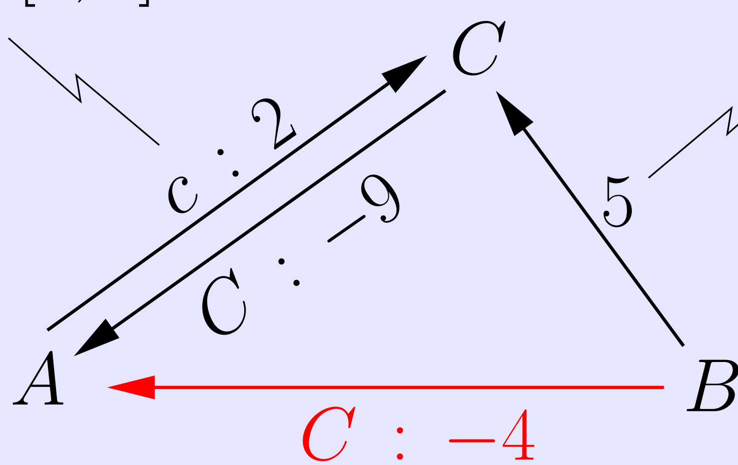
RTED Example (ctd.)

Contingent Link: $(A, 2, 9, C)$

$C - A \in [2, 9]$

$C - B \leq 5$

(i.e., $B \geq C - 5$)



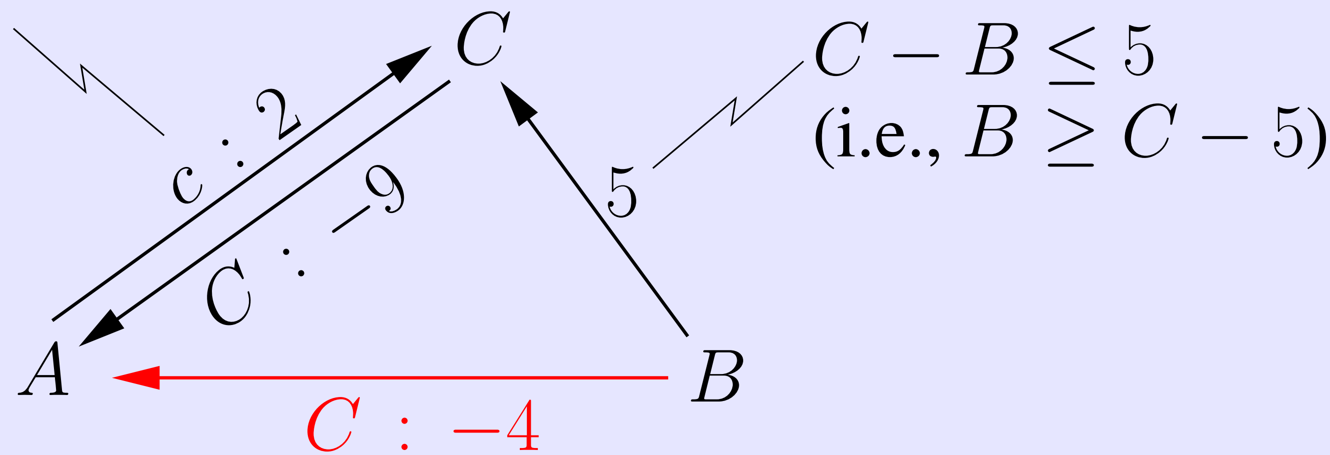
Initial Decision: $(4, \{B\})$

(If nothing happens before time 4, execute B at 4.)

RTED Example (ctd.)

Contingent Link: $(A, 2, 9, C)$

$C - A \in [2, 9]$



Possible Outcome: C does not execute yet;
so B is executed at 4

Next decision: WAIT (for C to execute)

Edge-Generation Rules

- *No Case* Rule
- *Upper-Case* Rule
- *Lower-Case* Rule
- *Cross-Case* Rule
- *Label-Removal* Rule

(Morris and Muscettola 2005)

The No-Case Rule

$$
\begin{tikzcd}[column sep=huge]
Q \arrow[r, "3"'] \arrow[rr, bend left, "\color{red}\mathbf{7}"] & S \arrow[r, "4"'] & T
\end{tikzcd}
$$

The Upper-Case Rule

$$\begin{tikzcd}[column sep=huge] Q \arrow[r, "3"'] \arrow[rr, bend left, "C\!:\!-7"] & C \arrow[r, "C\!:\!-10"'] & A \end{tikzcd}$$

The Lower-Case Rule

$$\begin{tikzcd}[column sep=huge] A \arrow[r, "c:3"'] \arrow[rr, bend left, "-2"] & C \arrow[r, "-5"'] & X \end{tikzcd}$$

(Applies since $-5 \leq 0$)

The Cross-Case Rule

$$\begin{tikzcd} A \arrow[r, "c:3"'] \arrow[rr, bend left, "C_2:-5"] & C \arrow[r, "C_2:-8"'] & A_2 \end{tikzcd}$$

(Applies since $-8 \leq 0$ and $C \not\equiv C_2$)

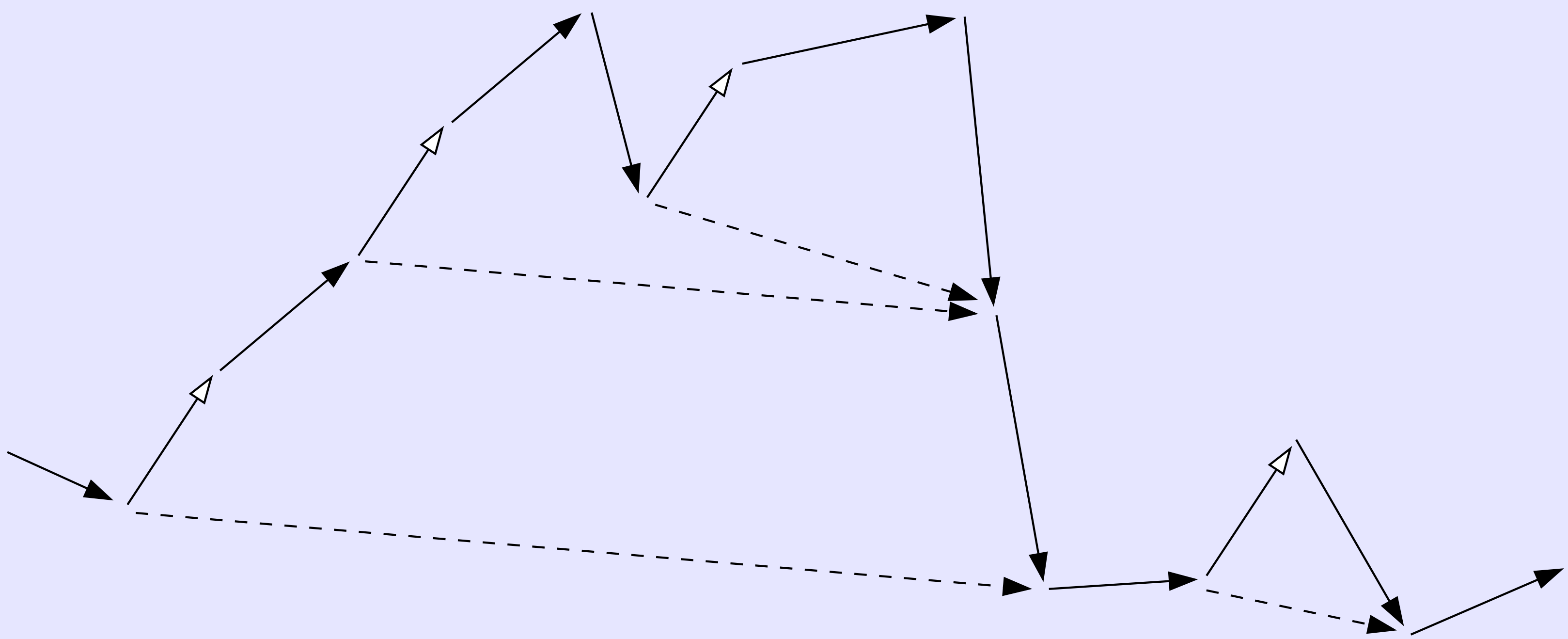
The Label-Removal Rule

$$\begin{tikzcd}[column sep=huge] X \arrow[r, "C:-1"'] \arrow[r, bend left, "-1", color=red] & A \arrow[r, "c:3"'] & C \end{tikzcd}$$

(Applies since $1 \leq 3$)

Semi-Reducibility

A path is *semi-reducible* if it can be transformed into a path with no *lower-case* edges.



Fundamental Theorem of STNUs

For an STNU $\mathcal{S}$, with graph $\mathcal{G}$, and APSSRP matrix $\mathcal{D}^*$, the following are equivalent:

- $\mathcal{S}$ is dynamically controllable
- $\mathcal{G}$ has no *semi-reducible* negative loops
- $\mathcal{D}^*$ has non-negative values on its main diagonal

(Morris and Muscettola 2005; Morris 2006; Hunsberger 2010; 2013b)

Flexible Execution of STNUs

- A DC STNU can be flexibly executed, incrementally computing updates using $O(N^2)$-time per execution event, $O(N^3)$-time overall.*

- As will be seen, this execution algorithm can be characterized as a dispatching algorithm for STNUs.

* (Hunsberger 2013a; 2015)

STNU Dispatchability

- For a DC STNU, Morris' $O(N^3)$-time DC-checking algorithm generates a dispatchable STNU.*

- Dispatchability same as for STNs, except that:
  - $\star$ contingent time-points are not controllable; and
  - $\star$ there are *wait* constraints: "As long as C unexecuted, X must wait at least 5 after A."

- Corollary: For a DC STNU, the STNU graph generated by exhaustively applying the constraint propagation rules from Morris et al. (2005) is dispatchable.

*(Morris 2014)

STNU Dispatchability (ctd.)

- Definition: A projection of an STNU is the STN that results from fixing the duration of each contingent link to one of its legal values.

- Definition: An STNU (including any wait constraints) is dispatchable if each of its STN *projections* is dispatchable (as an STN).

- Theorem: A dispatchable STNU is DC.*

* (Morris 2014)

STNU Summary

- The theory of STNUs (dynamic controllability, dispatchability, flexible execution) has been advanced dramatically over the past few years.
- Many important contributions from Paul Morris and colleagues.
- STNUs are ready for prime time!

Conditional STNs

Motivation for CSTNs

- Many actions generate information (e.g., medical tests, opening a box, monitoring traffic).

- The generated information is generally not known in advance, but discovered in real time.

- Some actions only make sense in certain scenarios (e.g., don't give drug if test result is negative).

- An execution strategy could be more flexible if it could react dynamically to generated information.

Motivation for CSTNs (ctd.)

- Many businesses using *workflow management systems* to automate manufacturing processes.
- Hospitals can use workflows to represent possible treatment pathways for a patient.
- CSTNs can serve as the temporal foundation for workflow management systems.

Conditional STNs (CSTNs)*

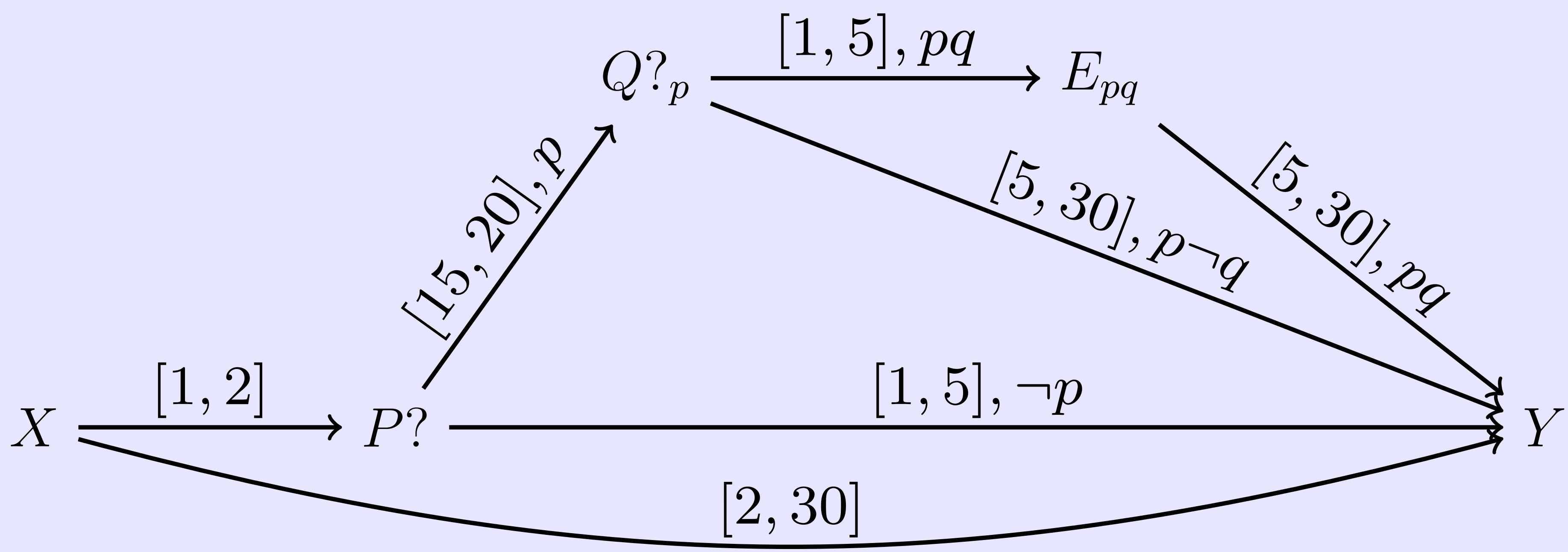
- Time-points and temporal constraints like in STNs
- Observation time-points generate truth values for propositional letters
- Time-points and constraints labeled by conjunctions of propositional letters

* (Tsamardinos, Vidal, and Pollack 2003)

Propositional Labels in CSTNs

- Propositional letters: $p, q, r, s, t, \ldots$.
- Each p has corresp. observation time-point, $P?$; executing $P?$ generates truth value for p.
- Label: conjunction of literals (e.g., $p(\neg q)r$).
- A scenario specifies values for *all* letters; the real scenario is only revealed incrementally.
- Time-points and constraints can be labeled; they only apply in scenarios where their labels are true.

Sample CSTN



$P?$ and $Q?$ represent tests for a patient.
$Q?$ is called a *child* of $P?$.

Dynamic Consistency of CSTNs

- Dynamic Execution Strategy: execution decisions may react to observations.
- A CSTN is *dynamically consistent* if there exists a dynamic execution strategy that guarantees that all *relevant* constraints will be satisfied no matter which scenario is incrementally revealed over time.

DC-Checking for CSTNs

- Convert to Disjunctive Temporal Network (Tsamardinos, Vidal, and Pollack 2003)
- Convert to Timed Game Automaton (Cimatti et al. 2014)
- Convert to Hyper Temporal Network (Comin and Rizzi 2015)
- Propagate labeled constraints (Hunsberger, Posenato, and Combi 2015)

DC Checking via Propagation

- Propagate *labeled* constraints
  — Motivated by related work (Conrad and Williams 2011)
- Introduce new kinds of literals and labels:
  Q-literals (e.g., $p?$) and *Q-labels* (e.g., $p\neg q(r?)s$)
- Address *negative q-loops* and *negative q-stars*

Labeled Constraints

$$X \xrightarrow{\quad\langle \delta, \ell \rangle\quad} Y$$

$Y - X \leq \delta$ must hold in scenarios where ℓ is true.

(If $\ell = \boxdot$, then $Y - X \leq \delta$ must hold in all scenarios.)

Propagation Rules for CSTNs

Labeled Propagation:	LP and qLP
Label Modification:	R_0 and qR_0
Label "Spreading":	R_3^* and qR_3^*

(The "q" rules propagate q-labeled constraints.)

The LP Rule

$$\begin{tikzcd} W \arrow[r, "\langle 3, pq \rangle"'] \arrow[rr, bend left, "\langle 8, pq\neg r \rangle", color=red] & X \arrow[r, "\langle 5, q\neg r \rangle"'] & Y \end{tikzcd}$$

Labels of two pre-existing edges are conjoined;
The resulting label must be consistent.

The R_0 Rule

$$\begin{tikzcd}
P? \arrow[rr, bend left, "{\langle -5, q\neg r\rangle}"] \arrow[rr, "{\langle -5, pq\neg r\rangle}"'] & & X
\end{tikzcd}$$

Edge weight must be negative;
Any occurrence of p (or $\neg p$) removed from label.

The R_3^* Rule

$$\begin{tikzcd}[column sep=huge]
P? \arrow[r, "{\langle -3, qr \rangle}"'] & X & Y \arrow[l, "{\langle -8, pqs \rangle}"'] \arrow[l, bend right, red, "{\color{red}\langle -3, qrs \rangle}"']
\end{tikzcd}$$

Pre-existing labels must be consistent;
Generated label is conjunction of pre-existing labels
— minus any occurrence of p (or $\neg p$);
Lefthand weight must be negative;
Generated weight is max of pre-existing weights.

Propagating Q-Labels

- Propagating along consistent labels insufficient
- *Q-labels*: contain literals such as $p?$. A constraint labeled by $p?$ must hold as long as p's value unknown.
- Conjunction operation expanded:
  $$p \wedge \neg p \equiv p?; \;\; p \wedge p? \equiv p?; \;\; \neg p \wedge p? \equiv p?; \;\; \text{etc.}$$
- Q-labels only needed on *lower-bound* constraints (i.e., edges pointing at Z).

The qLP Rule

$$\begin{tikzcd}[column sep=huge] W \arrow[r, "\langle -3, pq\rangle"'] \arrow[rr, bend left, "\langle -8, p(q?)\neg r\rangle"] & X \arrow[r, "\langle -5, \neg q \neg r\rangle"'] & Z \end{tikzcd}$$

Generated edge terminates at Z;
Labels need not be consistent;
Edge weights must be negative.

The qR_0 Rule

$$\begin{tikzcd} P? \arrow[rr, bend left, "{\langle -5, q\neg r\rangle}"] \arrow[rr, "{\langle -5, (p?)q\neg r\rangle}"'] & & Z \end{tikzcd}$$

Edge must terminate at Z;
Edge weight must be negative;
Any occurrence of p (or $\neg p$ or $p?$) removed from label.

The $\mathbf{qR}_3^*$ Rule

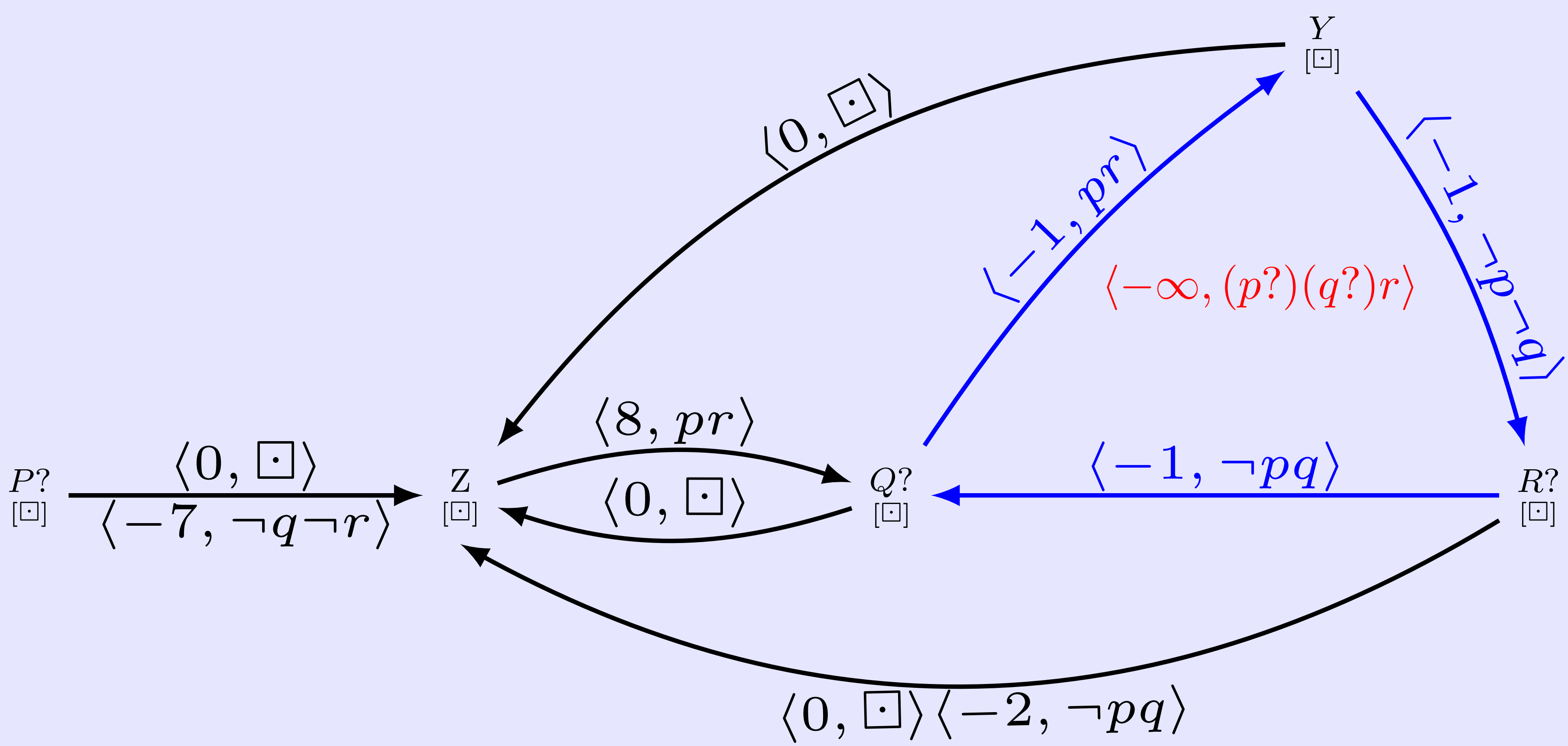
$$
\begin{tikzcd}
P? \arrow[r, "{\langle -3, q(r?) \rangle}"'] & Z & Y \arrow[l, "{\langle -8, p\neg qr(s?) \rangle}"] \arrow[l, bend right, red, "{\color{red}\langle -3, (q?)(r?)(s?) \rangle}"']
\end{tikzcd}
$$

Labels need not be consistent;

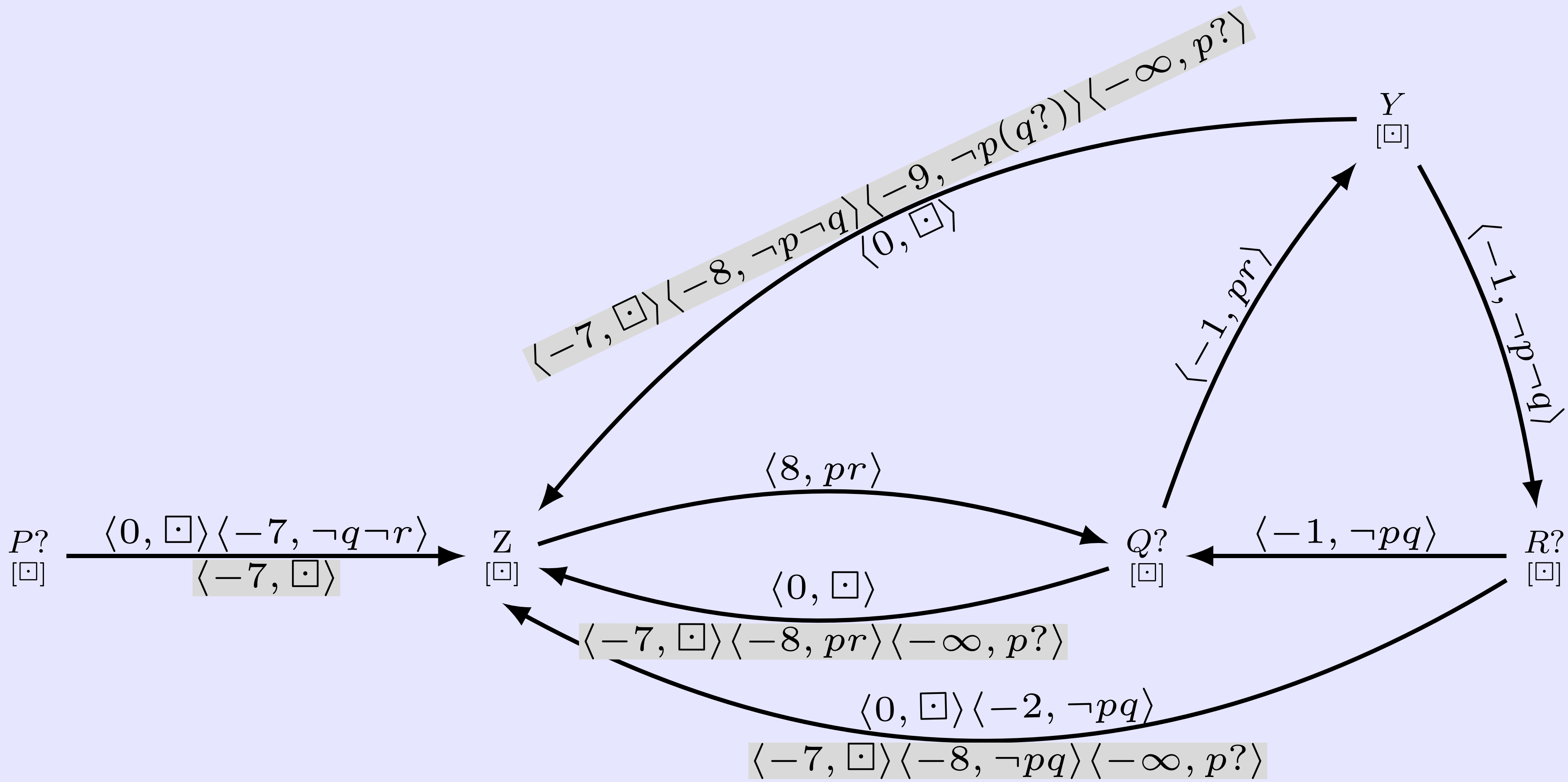
Lefthand weight must be negative;

Generated weight is max of pre-existing weights.

Negative Q-Loop Example

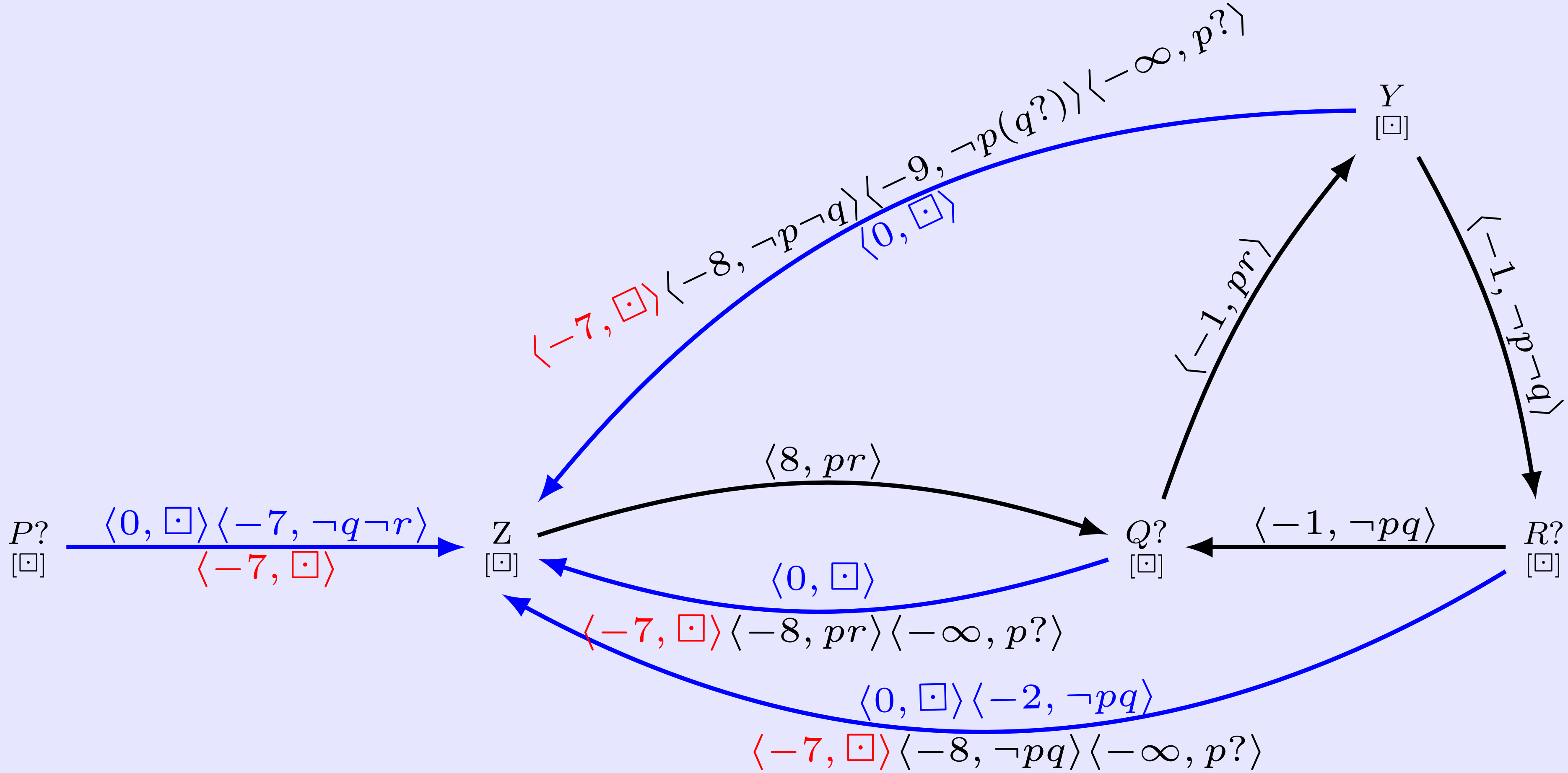


Completing the Propagation



The *Spreading Lemma*

The minimum lower-bound constraint $\langle -7, \boxdot\rangle$ has spread to all unexecuted time-points.



DC-Checking Alg. for CSTNs

- The DC-Checking Alg. does exhaustive propagation
- Returns `NO` if any negative loop with a consistent label is ever found; otherwise returns `YES`.
- In positive cases, constructs *earliest-first* strategy, which is viable due to the spreading lemma.
- Although exponential-time in the worst case, shown to be practical across a variety of sample networks.

(Hunsberger, Posenato, and Combi 2015)

The *Earliest-First* Strategy

- Keep track of *current partial scenario* (CPS), π.
  Initially $\pi = \boxdot$.

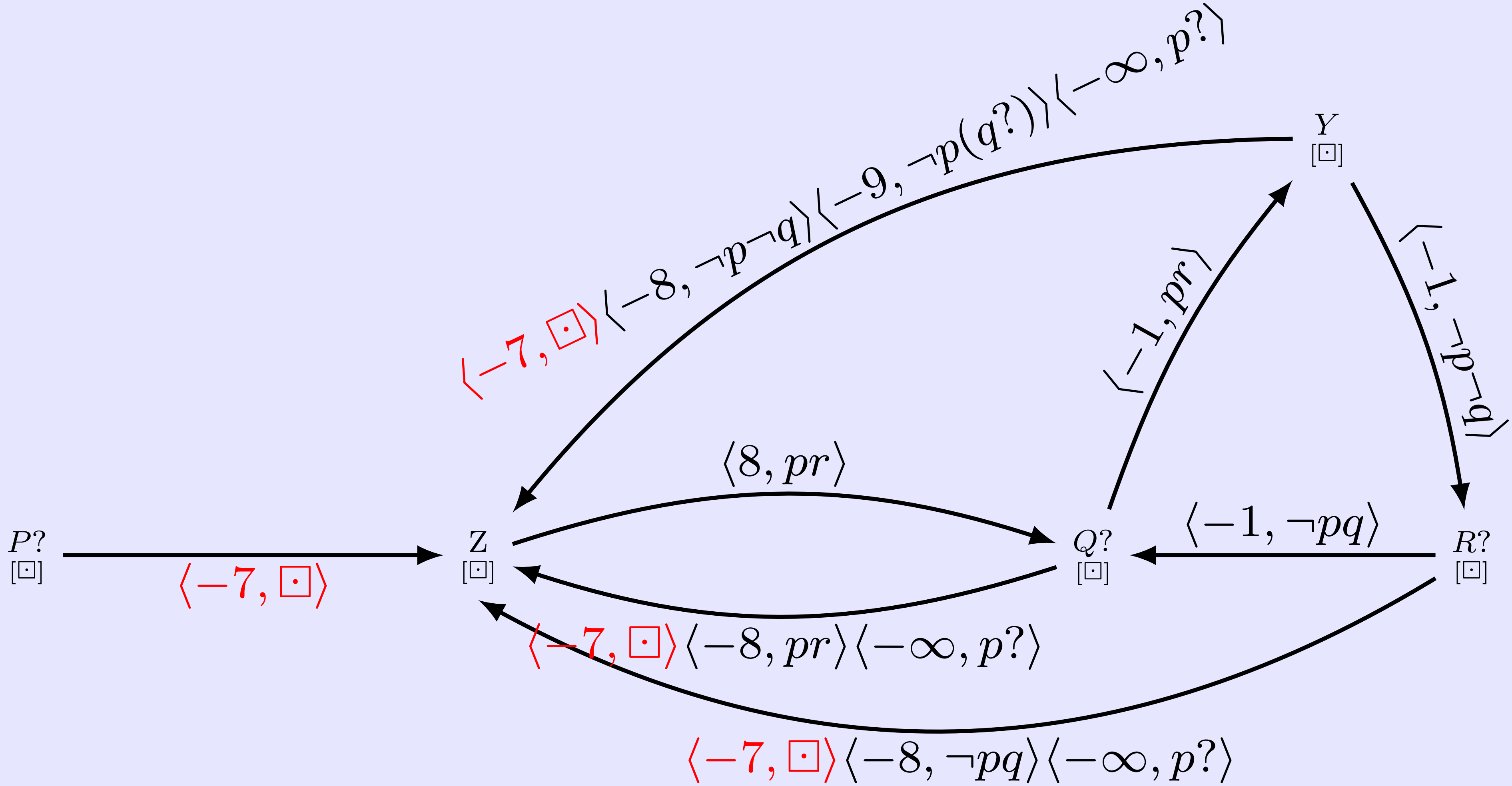
- After each execution event, compute *effective lower bound* (ELB) for each as-yet-unexecuted time-point.

- $ELB(X, \pi)$ restricts attention to lower bounds for X whose labels are applicable to π.

- Execute X next if it has the minimum ELB value.

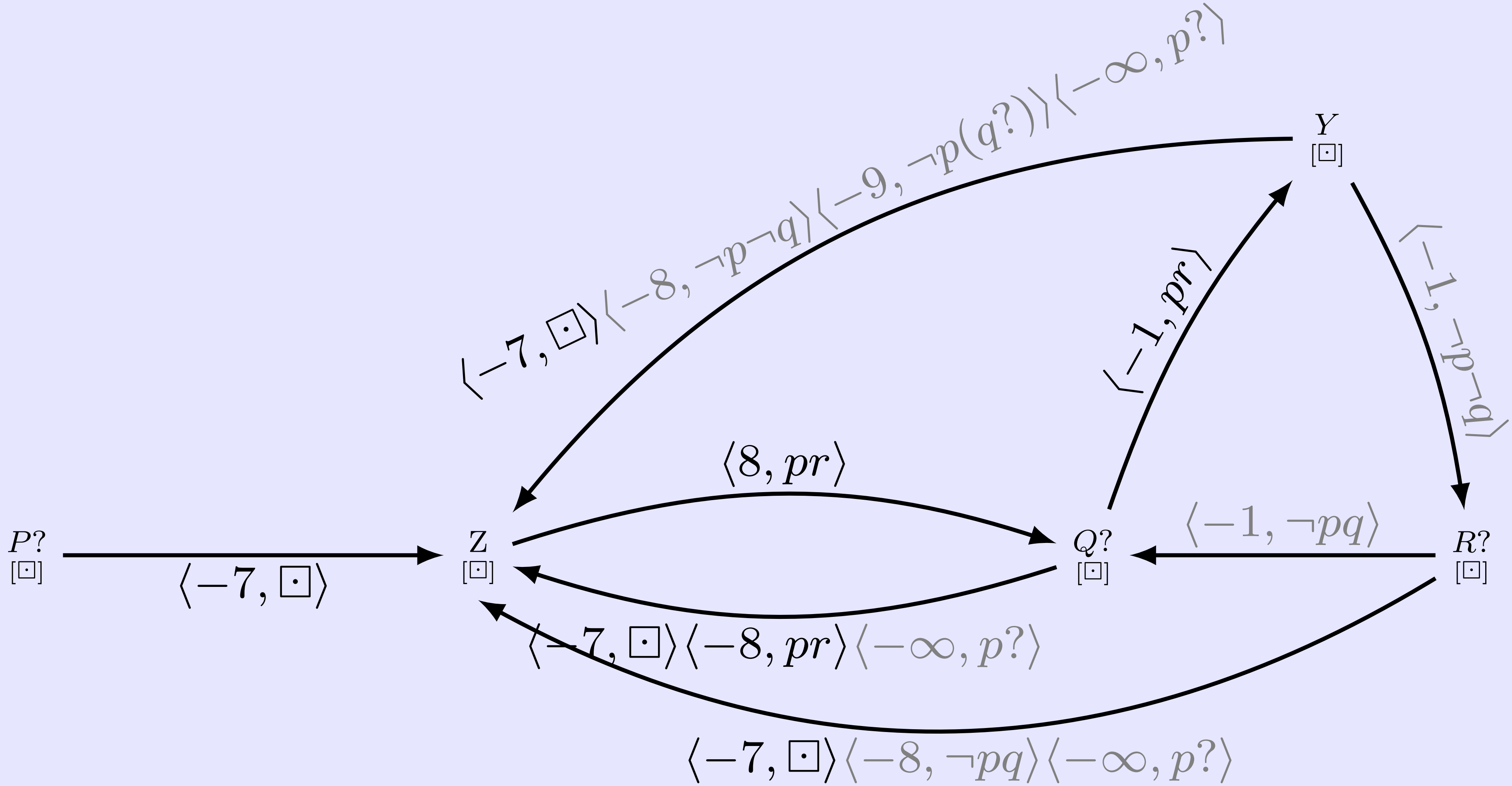
Sample Execution

$\pi = \boxdot, \quad Z = 0, \quad ELB(P?, \boxdot) = -7;\quad$ execute $P? = 7.$



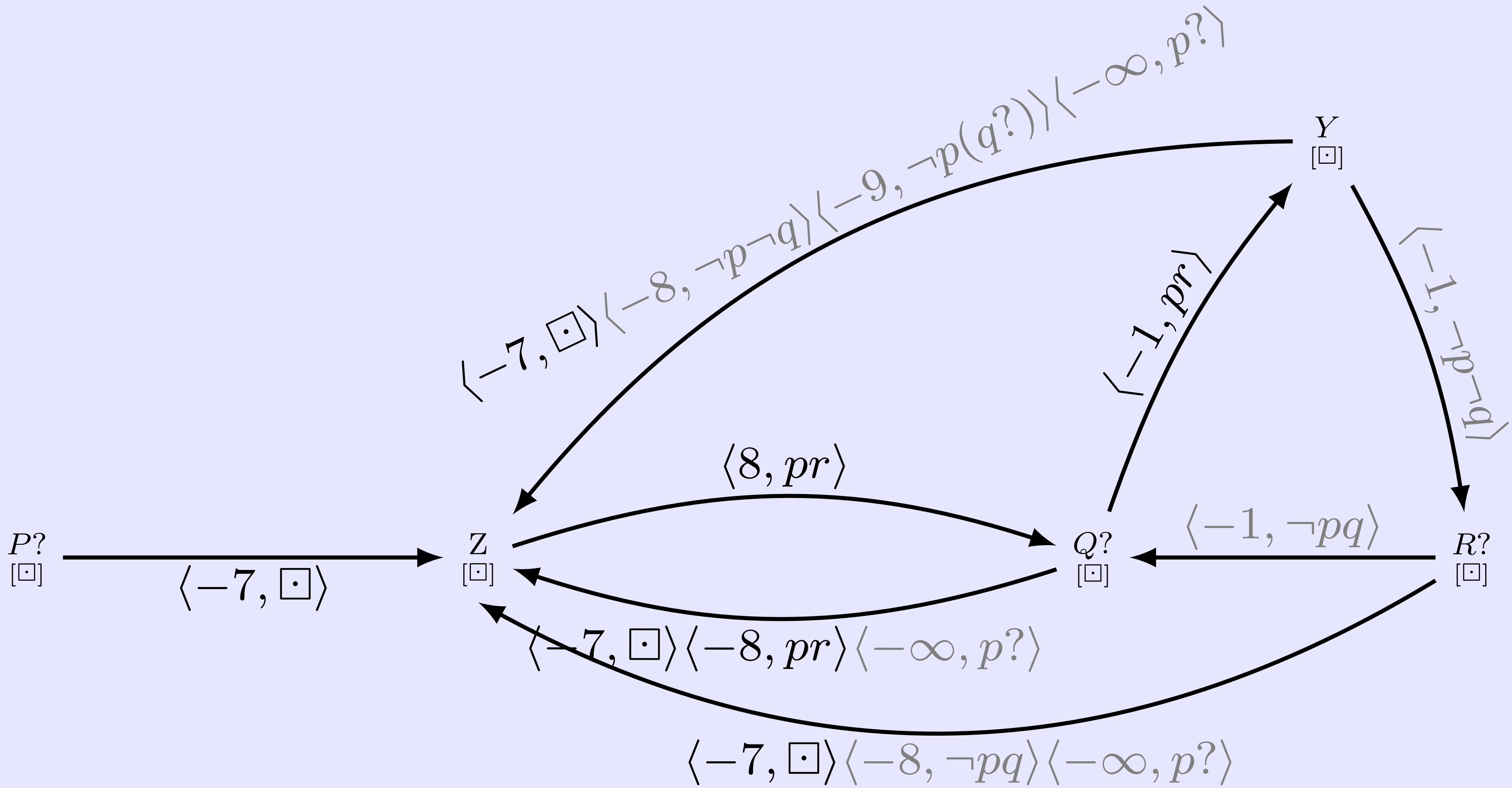
Sample Execution (ctd.)

Suppose $p = true$. $\pi = p$; $ELB(Y, p) = 7 = ELB(R?, p)$.
So execute $Y = 7$ and $R? = 7$.



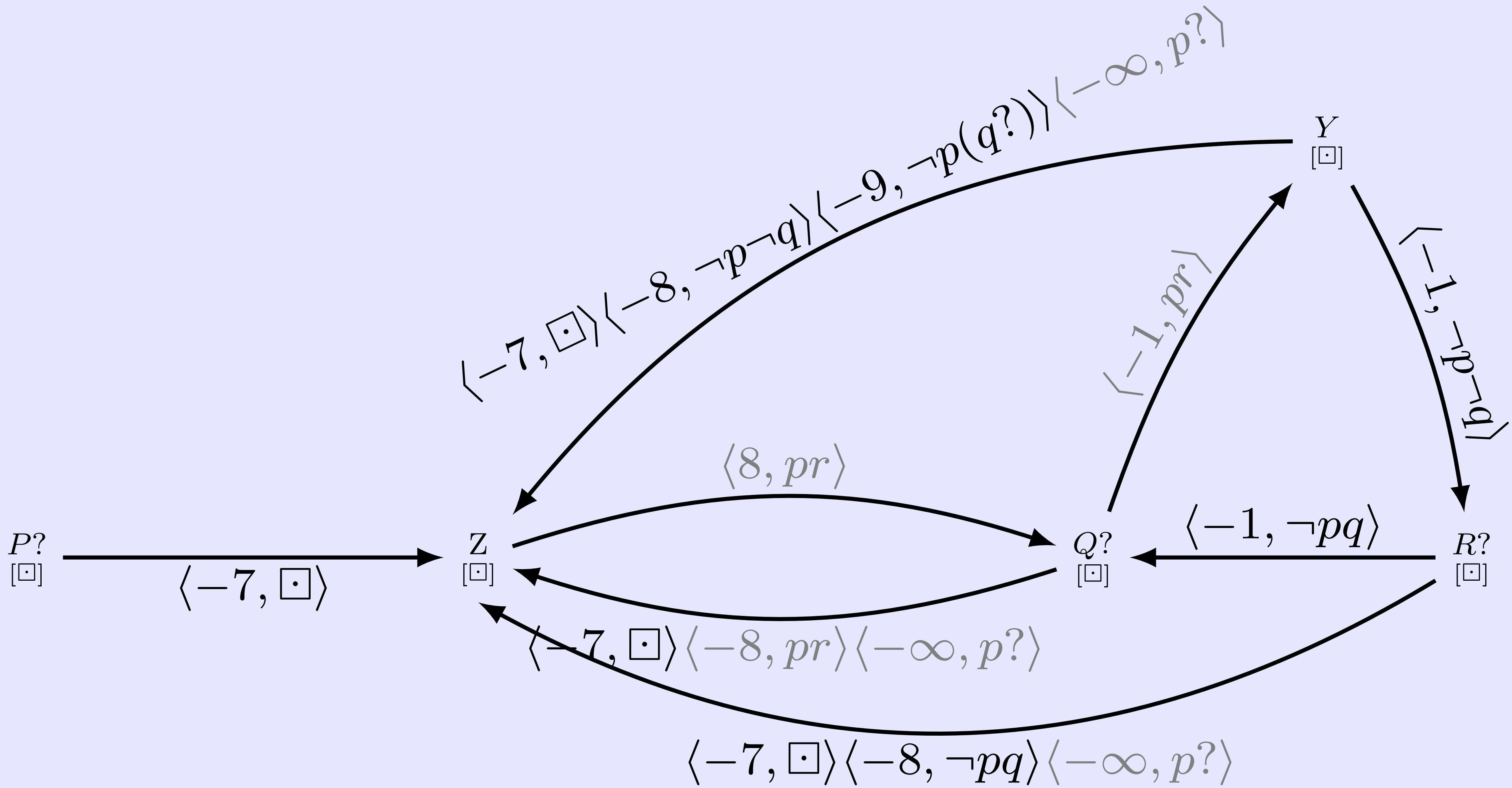
Sample Execution (ctd.)

Suppose $r = true$. $\pi = pr$; $ELB(Q?, p) = 8$.
So execute $Q? = 8$.



Alternative Execution

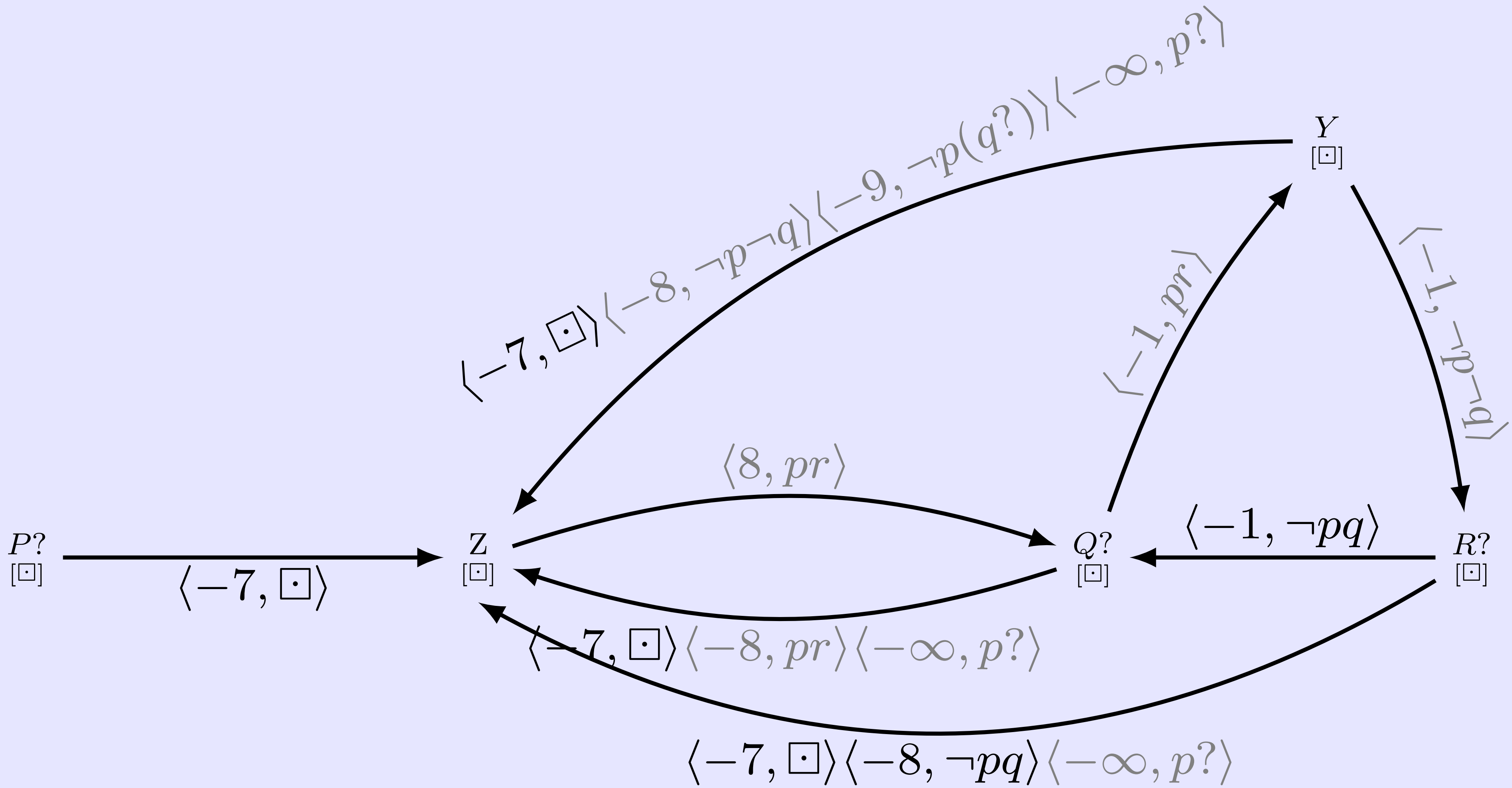
Suppose $p = false$. $\pi = \neg p$; $ELB(Q?, p) = 7$.
So execute $Q? = 7$.



Alternative Execution (ctd.)

Suppose $q = true$. $\pi = \neg pq$; $ELB(Y, \neg pq) = 7$.

So execute $Y = 7$. Afterward, execute $R? = 8$.



Bounded Reaction Time

- ϵ-dynamic controllability requires bounded reaction time $\epsilon > 0$ (Comin and Rizzi 2015).
- Propagation-based ϵ-DC checking algorithm (Hunsberger and Posenato 2016).
- Semantics of instantaneous reactivity for CSTNs (Cairo, Comin, and Rizzi 2016).

CSTN Summary

- Theory of dynamic consistency for CSTNs very solid (instantaneous/non-instantaneous reactivity; bounded reaction time).
- Several competing DC-checking algorithms—all are exponential, but propagation-based algorithm shows promise.
- More work to do on flexible execution.

CSTNUs

CSTNUs

- A *Conditional Simple Temporal Network with Uncertainty* (CSTNU) combines contingent links from STNUs and observation time-points from CSTNs.
- Sound-but-not-complete DC-checking algorithm presented years ago (Combi, Hunsberger, and Posenato 2013).
- Sound-and-complete DC-checking algorithm that extends rules for STNUs and CSTNs is forthcoming!

CDTNUs

Adding Disjunction to CSTNUs

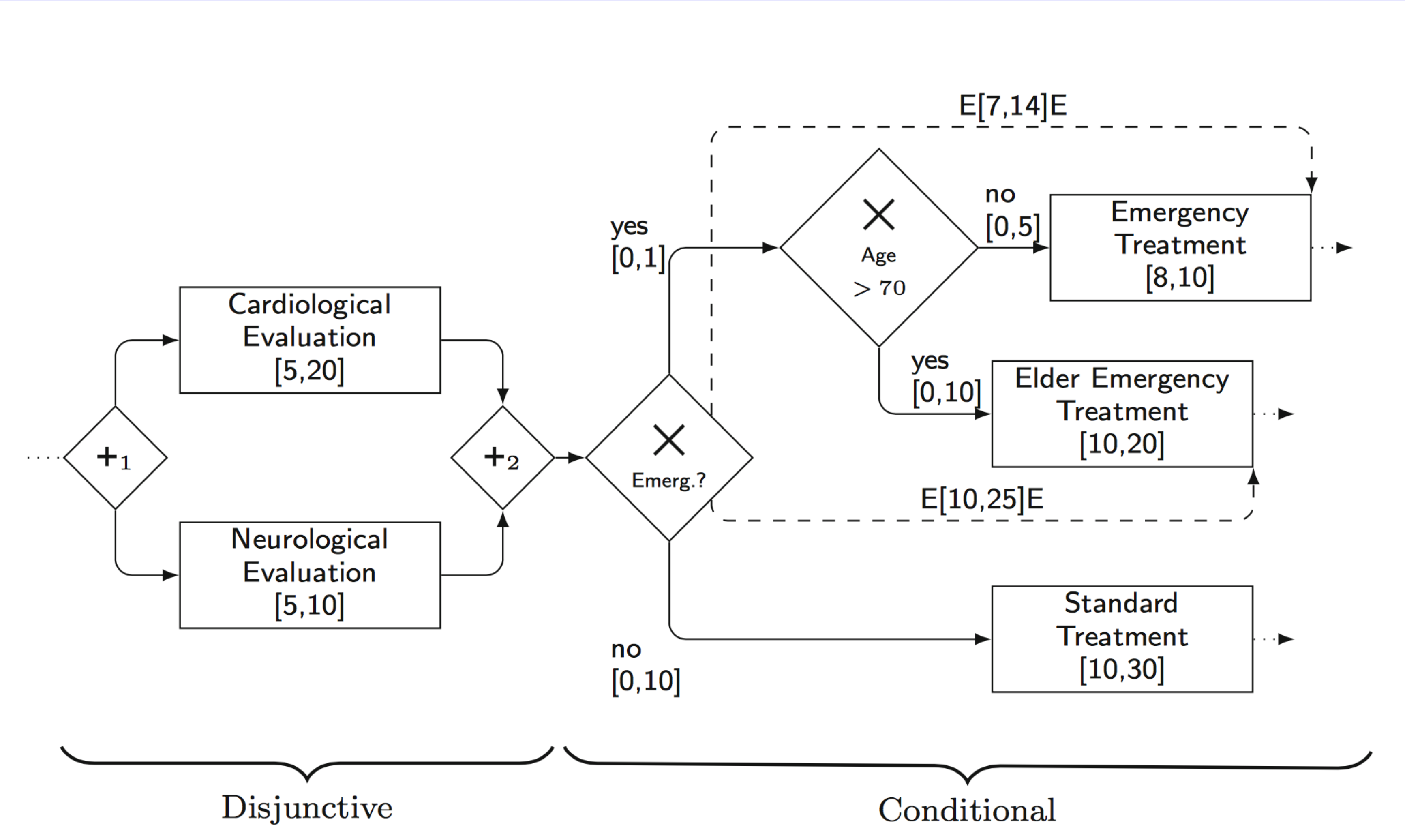
- A *Conditional Disjunctive Temporal Network with Uncertainty* (CDTNU) augments a CSTNU to include disjunctive constraints.

- Possible to convert the DC-checking problem for CDTNUs into a *controller-synthesis* problem for a *Timed Game Automaton* (TGA)*.

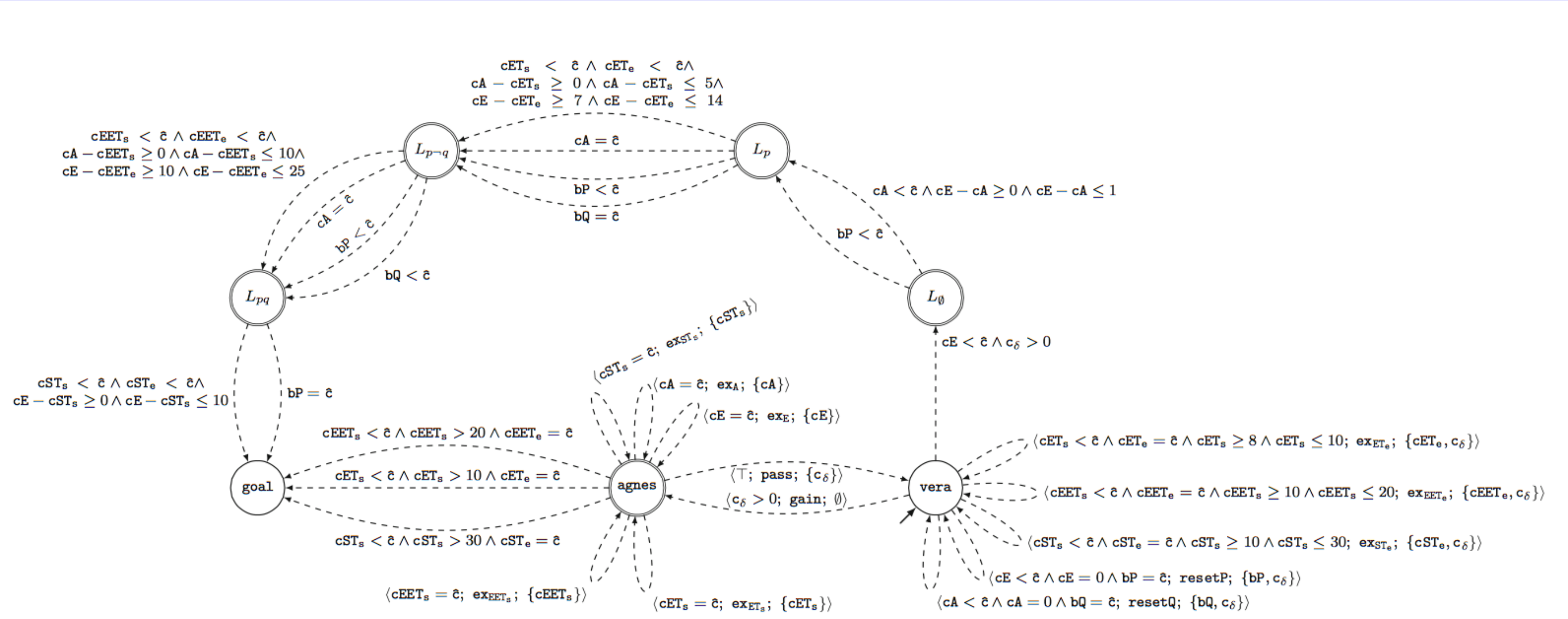
- Highlights connections between temporal networks and TGAs, but algorithm not yet practical.

* (Cimatti et al. 2016)

Sample Workflow



TGA Encoding of Workflow



Conclusions

Conclusions

- Theoretical foundations for a variety of temporal networks are quite solid.
- STNs have been incorporated into planning and scheduling applications for over a decade.
- $O(N^3)$-time DC-checking/dispatchability algorithm for STNUs makes them ready for prime time.
- Propagation-based algorithms for CSTNs and CSTNUs show promise.

References

- Cairo, M.; Comin, C.; and Rizzi, R. 2016. Instantaneous reaction-time in dynamic-consistency checking of conditional simple temporal networks. In *Proceedings of the 23rd International Symposium on Temporal Representation and Reasoning (TIME-2016)*.
- Cesta, A., and Oddi, A. 1996. Gaining efficiency and flexibility in the simple temporal problem. In *Proceedings of the Third International Workshop on Temporal Representation and Reasoning (TIME-96)*, 45–50. IEEE.
- Cimatti, A.; Hunsberger, L.; Micheli, A.; Posenato, R.; and Roveri, M. 2014. Sound and complete algorithms for checking the dynamic controllability of temporal networks with uncertainty, disjunction and observation. In *21st International Symposium on Temporal Representation and Reasoning (TIME-2014), Verona, Italy*, 27–36. IEEE Computer Society.
- Cimatti, A.; Hunsberger, L.; Micheli, A.; Posenato, R.; and Roveri, M. 2016. Dynamic controllability via timed game automata. *Acta Informatica* 53:681–722.
- Combi, C.; Hunsberger, L.; and Posenato, R. 2013. An algorithm for checking the dynamic controllability of a conditional simple temporal network with uncertainty. In Filipe, J., and Fred, A., eds., *Proceedings of the 5th International Conference on Agents and Artificial Intelligence (ICAART-2013)*, volume 2, 144–156. SCITEPRESS.
- Comin, C., and Rizzi, R. 2015. Dynamic consistency of conditional simple temporal networks via mean payoff games: a singly-exponential time dc-checking. In Grandi, F.; Lange, M.; and Lomuscio, A., eds., *22st International Symposium on Temporal Representation and Reasoning (TIME 2015)*, 19–28. IEEE CPS.
- Conrad, P. R., and Williams, B. C. 2011. Drake: An efficient executive for temporal plans with choice. *Journal of Artificial Intelligence Research (JAIR)* 42:607–659.
- Cormen, T. H.; Leiserson, C. E.; and Rivest, R. L. 1990. *Introduction to Algorithms*. Cambridge, MA: The MIT Press.
- Dechter, R.; Meiri, I.; and Pearl, J. 1991. Temporal constraint networks. *Artificial Intelligence* 49:61–95.
- Demetrescu, C., and Italiano, G. 2002. A new approach to dynamic all pairs shortest paths. Technical Report ALCOMFT-TR-02-92, ALCOM-FT. To appear in Proceedings of the 35th Annual ACM Symposium on Theory of Computing (STOC'03), San Diego, California, June 2003.
- Even, S., and Gazit, H. 1985. Updating distances in dynamic graphs. *Methods of Operations Research* 49:371–387.
- Gerevini, A.; Perini, A.; and Ricci, F. 1996. Incremental algorithms for managing temporal constraints. Technical Report IRST-9605-07, IRST.
- Hunsberger, L., and Posenato, R. 2016. Checking the dynamic consistency of conditional simple temporal networks with bounded

reaction times. In Coles, A.; Coles, A.; Edelkamp, S.; Magazzeni, D.; and Sanner, S., eds., *Proceedings of the 26th International Conference on Automated Planning and Scheduling (ICAPS-2016)*.

- Hunsberger, L.; Posenato, R.; and Combi, C. 2015. A sound-and-complete propagation-based algorithm for checking the dynamic consistency of conditional simple temporal networks. In Grandi, F.; Lange, M.; and Lomuscio, A., eds., *22st International Symposium on Temporal Representation and Reasoning (TIME 2015)*, 4–18. IEEE.
- Hunsberger, L. 2008. A practical temporal constraint management system for real-time applications. In *Proceedings of the European Conference on Artificial Intelligence (ECAI-2008)*, 553–557. IOS Press.
- Hunsberger, L. 2009. Fixing the semantics for dynamic controllability and providing a more practical characterization of dynamic execution strategies. In *Proceedings of the 16th International Symposium on Temporal Representation and Reasoning (TIME-2009)*, 155–162. IEEE Computer Society.
- Hunsberger, L. 2010. A fast incremental algorithm for managing the execution of dynamically controllable temporal networks. In *Proceedings of the 17th International Symposium on Temporal Representation and Reasoning (TIME-2010)*, 121–128. IEEE.
- Hunsberger, L. 2013a. A faster execution algorithm for dynamically controllable STNUs. In Sanchez, C.; Venable, K. B.; and Zimanyi, E., eds., *Proceedings of the 20th International Symposium on Temporal Representation and Reasoning (TIME-2013)*. IEEE Computer Society.
- Hunsberger, L. 2013b. Magic loops in simple temporal networks with uncertainty. In *Proceedings of the Fifth International Conference on Agents and Artificial Intelligence (ICAART-2013)*.
- Hunsberger, L. 2015. Efficient execution of dynamically controllable simple temporal networks with uncertainty. *Acta Informatica* 53(2):89–147.
- Morris, P. H., and Muscettola, N. 2005. Temporal dynamic controllability revisited. In *The Twentieth National Conference on Artificial Intelligence (AAAI-2005)*, 1193–1198. The MIT Press.
- Morris, P.; Muscettola, N.; and Vidal, T. 2001. Dynamic control of plans with temporal uncertainty. In *Seventeenth International Joint Conference on Artificial Intelligence (IJCAI-01)*, 494–499. Morgan Kaufmann.
- Morris, P. 2006. A structural characterization of temporal dynamic controllability. In *Principles and Practice of Constraint Programming (CP 2006)*, volume 4204 of *Lecture Notes in Computer Science*. Springer. 375–389.
- Morris, P. 2014. Dynamic controllability and dispatchability relationships. In H., S., ed., *Integration of AI and OR Techniques in Constraint Programming, CPAIOR 2014*, volume 8451 of *Lecture Notes in Computer Science*. Springer.
- Morris, P. 2016. The mathematics of dispatchability revisited. In Coles, A.; Coles, A.; Edelkamp, S.; Magazzeni, D.; and Sanner, S., eds., *Proceedings of the 26th International Conference on Automated Planning and Scheduling (ICAPS-2016)*.

- Muscettola, N.; Morris, P.; and Tsamardinos, I. 1998. Reformulating temporal plans for efficient execution. In *Proceedings of the Sixth International Conference on Principles of Knowledge Representation and Reasoning (KR-98)*.
- Ramalingam, G., and Reps, T. 1996. On the computational complexity of dynamic graph problems. *Theoretical Computer Science* 158:233–277.
- Ramalingam, G.; Song, J.; Joskowicz, L.; and Miller, R. 1999. Solving systems of difference constraints incrementally. *Algorithmica* 23:261–275.
- Rohnert, H. 1985. A dynamization of the all pairs least cost path problem. In Mehlhorn, K., ed., *2nd Symposium of Theoretical Aspects of Computer Science (STACS 85)*, volume 182 of *Lecture Notes in Computer Science*. Springer. 279–286.
- Tsamardinos, I.; Muscettola, N.; and Morris, P. 1998. Fast transformation of temporal plans for efficient execution. In *Proceedings of the Fifteenth National Conference on Artificial Intelligence (AAAI-98)*. Cambridge, MA: The MIT Press. 254–261.
- Tsamardinos, I.; Vidal, T.; and Pollack, M. E. 2003. CTP: A new constraint-based formalism for conditional, temporal planning. *Constraints* 8:365–388.